Precoding Designs in Multiuser Multicell Wireless Systems: Competition and Coordination

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1 Introduction and Motivation

2 Research Contributions
   - Downlink Beamforming for Power Minimization
   - Block-Diagonalization Precoding
   - Multicell MIMO Multiple-Access Channel (MIMO-MAC)
   - Multicell MIMO Broadcast Channel (MIMO-BC)

3 Conclusion
The Future of Wireless Communications

- Rapid increase in subscriptions to mobile broadband services
- Higher throughput, higher robustness, and better coverage

Figure courtesy of Informa Telecoms & Media.
Technical challenges: efficient utilization of the radio spectrum
MIMO Communications

$C = \log(1 + \text{SNR})$

$C \approx \min(M, N) \log(\text{SNR})$

- Multiple-input Multiple-Output (MIMO): using multiple transmit/receive antennas
- MIMO precoding (beamforming) → Higher spectral efficiency
MIMO Configurations

ICI = Inter-cell Interference
Coordinated Multipoint Transmission/Reception

- Cellular networks → inter-cell interference (ICI)
- Coordinated Multipoint Transmission/Reception (CoMP)
  - Coordinate simultaneous transmissions from multiple BSs to the MSs
  - Actively deal with the ICI by the means of MIMO precoding
- CoMP is a key technology for Long-Term Evolution (LTE)-Advanced

CoMP Modes

- Competition *versus* Coordination
- Interference Aware (IA) *versus* Interference Coordination (IC)
Challenges in CoMP Precoding Designs

A paradigm shift in precoding designs

*Independent per-cell approach to coordinated multicell approach*

- Large-scale and distributed multicell network
- CSI: difficult to acquire
- Limited backhaul links for control/signaling
- Nonconvex precoding design problems
Competition and Coordination in Multicell Wireless Systems

*Distributed strategies in precoding designs and ICI management*

- **Distributed algorithms** for precoding designs and ICI management
- **Local computation** with local CSI
- **Define and quantize** the control/signaling messages
- **Achievable optimality** *versus* computational complexity
Research Contributions

Develop low-complexity and distributed algorithms
Devise the structure of the CoMP precoders
Devise the message exchange mechanism
Expose new perspectives and understanding the interactions between the coordinated BSs in a CoMP system
Design criteria:
- Power minimization and Sum-rate maximization
- IA and IC
- Uplink and Downlink
Downlink Beamforming for Power Minimization: A Game-Theoretical Approach

- Beamforming design to minimize the transmit power at each BS
- Guaranteed SINR requirement at each MS

\[ \text{SINR}_{q_i} \geq \gamma_{q_i}^{\text{min}}, \ \forall q, \ \forall i \] (1)


**Downlink Beamforming Game (IA mode)**

- **SINR at a particular MS**
  \[
  \text{SINR}_{q_i} = \frac{\left| w_{q_i}^H h_{qq_i} \right|^2}{\sum_{j \neq i} \left| w_{q_j}^H h_{qq_i} \right|^2 + r_{-q_i}},
  \]
  where \( r_{-q_i} \): sum ICI + noise.

- **Each BS is aware of the ICI at its connected MSs and selfishly adjusts its precoders accordingly**

- **Multicell game** \( G_P = \left( \Omega, \{ P_q(W_{-q}) \}_{q \in \Omega}, \{ t_q(W_q) \}_{q \in \Omega} \right) \)
  - **Players** \( \Omega \): base-stations
  - **Utility function**: \( t_q(W_q) = \sum_{i=1}^{K} \| w_{q_i} \|^2 \)
  - **Set of admissible strategies**: \( \mathcal{P}_q(W_{-q}) = \{ \text{SINR}_i(W_q) \geq \gamma_i^{\text{min}}, \forall i \} \)

- **\( W^* = \{ W^*_q \}_{q=1}^Q \) is a Nash Equilibrium (NE) if**
  \[
  t_q(W^*_q) \leq t_q(W_q), \forall W_q \in \mathcal{P}_q(W^*_q), \forall q \in \Omega. \quad (2)
  \]
Study the NE of game $G_P$

Unique NE if it exists

→ Best response strategies proved to converge (standard functions)

Sufficient and necessary conditions: low ICI guarantees the NE’s existence

IC to obtain Pareto-optimality

To improve the NE’s efficiency, modify the utility function

$$s_q(W_q) = \sum_{i=1}^{K} \|w_{qi}\|^2 + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \pi_{qrj} \|W_q^H h_{qrj}\|^2,$$

where $\pi_{qrj}$: interference price charged on the ICI

The new game $G'_P$ is able to obtain a Pareto-optimal solution
• IA mode
• IA mode with pricing with partial inter-cell CSI
• IC mode: IA mode with the right pricing scheme and full inter-cell CSI
Competition versus Coordination

Prob. of Convergence

- IA Mode (C)
- IA Mode (C1)
- IC Mode
- IA Mode – Pricing

Transmit Power

- Average Power Consumption $P_{\text{total}}/\sigma^2$ (in dB)
- IA Mode
- IC Mode
- IA Mode – Pricing

Target SINRs: $\gamma_{q_i} = 10$ dB (dashed lines) and $\gamma_{q_i} = 0$ dB (solid lines)

Coordination $\rightarrow$ power savings, better coverage

Interference pricing $\rightarrow$ limit ICI $\rightarrow$ improve the efficiency of the game’s NE
Precoding design to maximize the sum-rate at each cell
Power constraint at each BS
Block-Diagonalization (BD) precoding: suppress intra-cell interference
BD-Dirty Paper Coding (BD-DPC): to further enhance the sum-rate performance

BD Precoding - Competition

- Multicell game $G_R = \left( \Omega, \{S_q\}_{q \in \Omega}, \{R_q(Q_q)\}_{q \in \Omega} \right)$
  - Players $\Omega$: base-stations
  - Utility function: $R_q(Q_q) = \sum_{i=1}^{K} \log |I + H_{qq_i}^H R_{-q_i}^{-1}(Q_{-q})H_{qq_i}Q_{q_i}|$
  - Set of admissible strategies
    $$S_q = \left\{Q_{q_i} : \sum_{i=1}^{K} \text{Tr} \{Q_{q_i}\} \leq P_q, \quad Q_{q_i} \succeq 0, \quad H_{qq_j}Q_{q_i} = 0, \forall j \neq i \right\}$$

- $(Q^*_q, Q^*_{-q})$ is a NE of game $G_R$ if
  $$R_q(Q^*_q, Q^*_{-q}) \geq R_q(Q_q, Q^*_{-q}) , \forall Q_q \in S_q, \forall q \in \Omega. \quad (3)$$

- Obtain closed-form best-response (water-filling) strategies by solving
  $$\max_{Q_{q_1}, \ldots, Q_{q_K}} R_q(Q_q, Q_{-q}) \quad (4)$$
  subject to $Q_{q_i} \in S_q, \forall i$.

- Convergence proved by the contraction mapping
- A NE is always existent and unique at low ICI
Joint weighted sum-rate maximization

\[
\text{maximize } \sum_{q=1}^{Q} \omega_q R_q(Q_q, Q_{-q}) \\
\text{subject to } Q_{qi} \in S_q, \forall i, \forall q.
\]  \hspace{1cm} (5)

Nonconvex \rightarrow \text{iterative linear approximation (ILA) to } Q \text{ per-cell convex problems}

\[
\text{maximize } \sum_{i=1}^{K} \text{Tr}\{A_{qi} Q_{qi}\} \\
\text{subject to } Q_{qi} \in S_q, \forall i, \forall q.
\]  \hspace{1cm} (6)

where \( A_{qi} \): interference price charged on the ICI

Monotonic convergence to a local maximum (Gauss-Seidel update proved, Jacobi update observed)

Distributed implementation with message exchange between BSs to compute the price \( A_{qi} \)'s
Convergence

**Competition**

- **Solid lines**: BD & **Dashed lines**: BD-DPC

- **BD-DPC** → higher sum-rate than BD

- Coordination → higher sum-rate than competition

- Coordination → slower to converge than competition
**Network Sum-rate**

*versus BS-MS Distance*

- DPC > BD-DPC > BD
- Coordination >> Competition, especially at high ICI region
- Performance saturated at high ICI region with competition
  → Coordination

*versus Transmit Power*
Successive interference cancelation (SIC) on a per-cell basis
Precoding design to maximize the weighted sum-rate in the uplink

\[
\begin{align*}
\text{maximize} & \quad \sum_{q=1}^{Q} \omega_q \log \left| I + \mathbf{R}_q^{-1} \left( \sum_{i=1}^{K} \mathbf{H}_{qqi} \mathbf{X}_{qi} \mathbf{H}_{qqi}^H \right) \right| \\
\text{subject to} & \quad \text{Tr} \{ \mathbf{X}_{qi} \} \leq P_{qi}, \forall i, \forall q \\
& \quad \mathbf{X}_{qi} \succeq 0, \forall i, \forall q.
\end{align*}
\]  


ILA Solution Approach

- Approximation and decomposition into $Q$ per-cell outer problems

$$\max_{X_{q_1}, \ldots, X_{q_K}} \omega_q \log |R_q + \sum_{i=1}^{K} H_{qq_i} X_{q_i} H_{qq_i}^H| - \sum_{i=1}^{K} \text{Tr}\{A_{q_i} X_{q_i}\}$$

subject to $\text{Tr}\{X_{q_i}\} \leq P_{q_i}, \forall i$

$$X_{q_i} \succeq 0,$$

where $A_{q_i}$: interference pricing charged on the ICI.

- Further decomposition into $K$ per-user inner problems

$$\max_{X_{q_i}} \omega_q \log |I + R_{q_i}^{-1} H_{qq_i} X_{q_i} H_{qq_i}^H| - \text{Tr}\{A_{q_i} X_{q_i}\}$$

subject to $\text{Tr}\{X_{q_i}\} \leq P_{q_i}, X_{q_i} \succeq 0,$

where $R_{q_i} = R_q + \sum_{j \neq i}^{K} H_{qq_j} X_{q_j} H_{qq_j}^H$.

- Monotonic convergence to a local maximum (Gauss-Seidel update proved, Jacobi update observed)

- Distributed implementation with message exchange between BSs to compute the price $A_{q_i}$'s
WMMSE Solution Approach

- Transform the original nonconvex problem into a matrix weighted sum-MSE minimization problem (WMMSE)

\[
\begin{align*}
\text{minimize} & \quad \sum_{q=1}^{Q} \sum_{i=1}^{K} \omega_q \left[ \text{Tr} \left\{ W_{qi} E_{qi} \right\} - \log | W_{qi} | \right] \\
\text{subject to} & \quad \text{Tr} \{ V_{qi} V_{qi}^H \} \leq P_{qi}, \forall q, \forall i,
\end{align*}
\] (10)

where \( V_{qi} \): transmit beamformer, \( U_{qi} \): receive beamformer, \( W_{qi} \): weight matrix, and \( E_{qi} \): MSE matrix.

- Not jointly convex, but convex in each set of variables \( V_{qi}, U_{qi}, W_{qi} \)
- Iterative update across each set of variables in closed-form solutions
- Distributed implementation with monotonic convergence to a local optimum proved (also a local optimum of the original problem)
Monotonic convergence for both ILA and WMMSE algorithms
ILA: Jacobi (simultaneous) update converges faster than Gauss-Seidel (sequential) update
ILA converges faster than WMMSE
Coordination → higher network sum-rate than competition
**Network Sum-rate**

### Full Convergence

- Full convergence: $\text{ILA} \approx \text{WMMSE} > \text{Competition}$
- Limited iteration: $\text{ILA} > \text{WMMSE} > \text{Competition}$
- Coordination: require $\text{BS} \leftrightarrow \text{BS}$ signaling
Dirty Paper Coding (DPC) on a per-cell basis
Precoding design to maximize the weighted sum-rate in the downlink

\[
\begin{align*}
\text{maximize} & \quad \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \log \left| R_{qi} + H_{qqi} \left( \sum_{j=1}^{i} Q_{qj} \right) H_{qqi}^H \right| \\
\text{subject to} & \quad \sum_{i=1}^{K} \text{Tr}\{Q_{qi}\} \leq P_q, \forall q; \quad Q_{qi} \succeq 0, \forall i, \forall q.
\end{align*}
\] (11)


ILA Solution Approaches

Approximation and decomposition into $Q$ per-cell outer problems

\[
\text{maximize } \quad \omega_q \sum_{i=1}^{K} \log \left| \frac{R_{qi} + H_{qqi} \left( \sum_{j=1}^{i} Q_{qj} \right) H_{qqi}^{H}}{R_{qi} + H_{qqi} \left( \sum_{j=1}^{i-1} Q_{qj} \right) H_{qqi}^{H}} \right| - \sum_{i=1}^{K} \text{Tr}\{A_q Q_{qi}\} \quad (12)
\]

subject to \[ \sum_{i=1}^{K} \text{Tr}\{Q_{qi}\} \leq P_q, \quad Q_{qi} \succeq 0, \quad \forall i, \]

where $A_q$: interference pricing charged on the ICI.

Still a nonconvex problem

Connection to the MAC problem via the uplink-downlink duality

\[
\text{maximize } \quad \omega_q \log \left| I + \sum_{i=1}^{K} \tilde{H}_{qqi}^{H} X_{qi} \tilde{H}_{qqi} \right| - \sum_{i=1}^{K} \text{Tr}\{X_{qi}\} \quad (13)
\]

subject to \[ X_{qi} \succeq 0, \quad \forall i, \]

where $\tilde{H}_{qqi} = R_{qi}^{-1/2} H_{qqi} (A_q + \lambda I)^{-1/2}, \lambda$: Lagrangian multiplier.
ILA Solution Approaches

- Convergence and global optimality of the outer problem proved
- Distributed implementation with monotonic convergence to a local optimum proved (also a local optimum of the original problem)

WMMSE algorithm
  - Similar to the multicell MIMO-MAC
  - Transform to an equivalent WMMSE problem
  - Iterative updates to each set of variables
**Convergence**

- Convergence to the globally optimal solution of the outer problem
- Monotonic convergence for both ILA and WMMSE algorithms
- ILA: Jacobi (simultaneous) update converges faster than Gauss-Seidel (sequential) update
- Coordination → **higher network sum-rate** than competition
**Network Sum-rate**

- **Full Convergence**: ILA $\approx$ WMMSE $>$ Competition
- **Limited iteration**: ILA $>$ WMMSE $>$ Competition
- **Nonlinear precoding (DPC)** $>$ **Linear precoding**
Conclusion and Q&A

- Multicell multiuser MIMO to improve spectral efficiency
- Multicell coordination → power savings, sum-rate enhancement over multicell competition
- Precoding with multicell coordination: more complex, more signaling
- Various precoding techniques: linear MMSE, BD and BD-DPC, MAC with SIC, BC with DPC

Q&A