Chapter 13

RLC Circuits and Resonance
Impedance of Series RLC Circuits

- A series RLC circuit contains both inductance and capacitance.
- Since $X_L$ and $X_C$ have opposite effects on the circuit phase angle, the total reactance ($X_{tot}$) is less than either individual reactance.
Impedance of Series RLC Circuits

- When $X_L > X_C$, the circuit is predominantly inductive
- When $X_C > X_L$, the circuit is predominantly capacitive
- Total reactance $|X_L - X_C|$
- Total impedance for a series RLC circuit is:
  \[ Z_{\text{tot}} = \sqrt{R^2 + X_{\text{tot}}^2} \]
  \[ \theta = \tan^{-1}(X_{\text{tot}}/R) \]
Analysis of Series RLC Circuits

- A series RLC circuit is:
- Capacitive when $X_C > X_L$
- Inductive when $X_L > X_C$
- Resonant when $X_C = X_L$
- At resonance $Z_{tot} = R$
- $X_L$ is a straight line:
  \[ y = mx + b \]
- $X_C$ is a hyperbola:
  \[ xy = k \]
Series RLC impedance as a function of frequency.

Graph Including $Z_{tot} = R$ at Resonance
Voltage Across the Series Combination of L and C

- In a series RLC circuit, the capacitor voltage and the inductor voltage are always 180° out of phase with each other.
- Because they are 180° out of phase, $V_C$ and $V_L$ subtract from each other.
- The voltage across L and C combined is always less than the larger individual voltage across either element.
The voltage across the series combination of $C$ and $L$ is always less than the larger individual voltage across either $C$ or $L$.
Inductor voltage and capacitor voltage effectively subtract because they are out of phase.
Series Resonance

- Resonance is a condition in a series RLC circuit in which the capacitive and inductive reactances are equal in magnitude.
- The result is a purely resistive impedance.
- The formula for series resonance is:

\[
\frac{1}{2\pi\sqrt{LC}}
\]
At the resonant frequency \( (f_r) \), the reactances are equal in magnitude and effectively cancel, leaving \( Z_r = R \).
At the resonant frequency, $f_r$, the voltages across $C$ and $L$ are equal in magnitude. Since they are $180^\circ$ out of phase with each other, they cancel, leaving 0 V across the $CL$ combination (point $A$ to point $B$). The section of the circuit from $A$ to $B$ effectively looks like a short at resonance (neglecting winding resistance).
An illustration of how the voltage and current amplitudes respond in a series RLC circuit as the frequency is increased from below to above its resonant value. The source voltage is held at a constant amplitude.

**Frequency below Resonance**

\[ X_c > X_L \]

(a) As frequency is increased below resonance from 0: \( X_c > X_L \), \( I \) increases from 0, \( V_R \) increases from 0, \( V_c \) increases from \( V_{in} \), \( V_L \) increases from 0, and \( V_{CI} \) decreases from \( V_{in} \).

**Frequency at Resonance**

\[ X_c = X_L \]

(b) At the resonant frequency, \( X_c = X_L \).

**Frequency above Resonance**

\[ X_c < X_L \]

(c) As frequency is increased above resonance: \( X_c < X_L \), \( I \) decreases from \( V_{in} \), \( V_R \) decreases from \( V_{in} \), \( V_c \) decreases from \( (V_{in}/R)X_c \), \( V_L \) decreases from \( (V_{in}/R)X_L \), and \( V_{CI} \) increases from 0.
Generalized current and voltage magnitudes as a function of frequency in a series $RLC$ circuit. $V_C$ and $V_L$ can be much larger than the source voltage. The shapes of the graphs depend on particular circuit values.
Phase Angle of a Series RLC Circuit

Capacitive = ICE

Inductive = ELI
A basic series resonant band-pass filter
Bandwidth of Series Resonant Circuits

- Current is maximum at resonant frequency
- Bandwidth (BW) is the range between two cutoff frequencies \( f_1 \) to \( f_2 \)
- Within the bandwidth frequencies, the current is greater than 70.7% of the highest resonant value
The 70.7% cutoff point is also referred to as:

• The Half Power Point
• -3dB Point
Formula for Bandwidth

• Bandwidth for either series or parallel resonant circuits is the range of frequencies between the upper and lower cutoff frequencies for which the response curve (I or Z) is 0.707 of the maximum value.

\[ \text{BW} = f_2 - f_1 \]

• Ideally the center frequency is:

\[ f_r = (f_1 + f_2)/2 \]
Example of the frequency response of a series resonant band-pass filter with the input voltage at a constant 10 V rms. The winding resistance of the coil is neglected.

(a) As the frequency increases to \( f_1 \), \( V_{\text{out}} \) increases to 7.07 V.

(b) As the frequency increases from \( f_1 \) to \( f_r \), \( V_{\text{out}} \) increases from 7.07 V to 10 V.

(c) As the frequency increases from \( f_r \) to \( f_2 \), \( V_{\text{out}} \) decreases from 10 V to 7.07 V.

(d) As the frequency increases above \( f_2 \), \( V_{\text{out}} \) decreases below 7.07 V.
Selectivity

- Selectivity defines how well a resonant circuit responds to a certain frequency and discriminates against all other frequencies.
- The narrower the bandwidth steeper the slope, the greater the selectivity.
- This is related to the Quality (Q) Factor (performance) of the inductor at resonance. A higher Q Factor produces a tighter bandwidth.
  - \( Q = \frac{X_L}{(R + R_{\text{windings}})} \)
  - Bandwidth = \( \frac{F_r}{Q} \)
FIGURE 13-23  Comparative selectivity curves.
A basic series resonant band-stop filter
Generalized response curve for a band-stop filter
Example of the frequency response of a series resonant band-stop filter with $V_{in}$ at a constant 10 V rms. The winding resistance is neglected.

(a) As frequency increases to $f_1$, $V_{out}$ decreases from 10 V to 7.07 V.

(b) As frequency increases from $f_1$ to $f_r$, $V_{out}$ decreases from 7.07 V to 0 V.

(c) As frequency increases from $f_r$ to $f_2$, $V_{out}$ increases from 0 V to 7.07 V.

(d) As frequency increases above $f_2$, $V_{out}$ increases toward 10 V.
Parallel RLC Circuits - Skip
Tank Circuit

- A parallel resonant circuit stores energy in the magnetic field of the coil and the electric field of the capacitor. The energy is transferred back and forth between the coil and capacitor.
Parallel Resonant Circuits

• For parallel resonant circuits, the impedance is maximum (in theory, infinite) at the resonant frequency
• Total current is minimum at the resonant frequency
• Bandwidth is the same as for the series resonant circuit; the critical frequency impedances are at $0.707Z_{\text{max}}$
A basic parallel resonant band-pass filter
Generalized frequency response curves for a parallel resonant band-pass filter

- **AHA:**
  - Half Power
  - -3dB

![Graphs showing the frequency response curves with key points and labels.](image)
Example of the response of a parallel resonant band-pass filter with the input voltage at a constant 10 V rms
A basic parallel resonant band-stop filter
A simplified portion of a TV receiver showing filter usage

Passing Audio Frequency Carrier (4.5 MHz)

Blocking Audio Frequency Carrier (4.5 MHz)
A simplified diagram of a superheterodyne AM radio broadcast receiver showing the application of tuned resonant circuits.

Intermediate Frequency (IF) is:
- Local oscillator frequency
- Carrier Frequency

- 1055Khz
- 600 KHz
- 455 KHz

Tank Circuits used as Oscillators

Thomas L. Floyd
Electronics Fundamentals, 6e
Electric Circuit Fundamentals, 6e