

On the Sum-Rate of BICM-ID Transmission Over Vector-Perturbation Precoding in Multi-User Downlink

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Abstract—This paper proposes a channel code for vector-perturbation (VP) precoded transmission in multi-user downlink and examines its achievable sum-rate performance. In particular, we first find the most suitable outer convolutional code for VP precoded transmission under bit-interleaved coded-modulation with iterative decoding (BICM-ID) by applying a semi-analytic technique based on extrinsic information transfer charts. We then study the achievable sum-rate of the proposed BICM-ID based VP precoded system and compare it with the sum-capacity of dirty paper coding (DPC) scheme. Our investigation shows that under perfect channel state information (CSI), the sum-rate of the proposed system grows linearly over the number of users, which is the same growth of the DPC. Under quantized CSI and finite-rate feedback, a linear increase in feedback overhead per user is necessary to maintain such a linear sum-rate growth.

Index Terms—Bit interleaved coded modulation, iterative decoding, vector-perturbation, multi-user downlink, MIMO precoding, non-linear precoding, EXIT chart, multi-user MIMO

I. INTRODUCTION

Bit-interleaved coded-modulation with iterative decoding (BICM-ID) is a bandwidth-efficient coded-modulation technique that enables point-to-point single-input single-output and multiple-input multiple-output systems to operate near their capacity limits [1]–[3]. On the other hand, the studies for BICM-ID in point-to-multipoint transmissions, such as multi-user downlink, are very limited in literature and its performance is not well-known. This observation motivates us to investigate the implementation and performance of BICM-ID transmissions for such a multi-user transmission scenario. It is well-known that the capacity limit of a multi-user downlink system is achievable by dirty-paper coding (DPC) [4], [5]. The sum-capacity of multi-user downlink achievable by DPC shows a linear growth with the minimum of the number of base-station (BS) antennas M and the number of single-antenna users K , i.e., $\min(M, K)$ [6], [7]. However, DPC is too complex for practical implementation and thus

only serves as a theoretical benchmark performance criterion. The combination of a suitable channel code such as turbo or turbo-like codes over non-linear vector-perturbation (VP) precoding has been considered as a more realistic alternative to DPC approach [8]–[12]. Different from these works, this paper studies VP precoded transmission under the BICM-ID framework and compares the achievable sum-rate performance of the proposed system against the DPC benchmark.

In VP precoding, there are typically two approaches: conventional VP and minimum mean-squared error VP (MMSE-VP). In conventional VP, for a given linear precoder, a perturbation vector v is added to user data symbol vector u such that the power scaling factor, denoted as γ , is minimized [7], [10], [13]. In MMSE-VP, the both perturbation vector v and the precoder are jointly optimized to minimize the end-to-end mean-squared-error [9]–[12]. While the conventional VP aims to improve the signal-to-noise ratio (SNR) for users, the MMSE-VP pursues a balance between the residual interference mitigation and the noise enhancement suppression [11], [12]. As a result, MMSE-VP precoded (uncoded) system is shown to have better bit error rate (BER) performance than the conventional VP, Tomlinson-Harashima, zero-forcing (ZF) and regularized-ZF precoding schemes [7], [9]–[11], [13]. However, to the best of our knowledge, a suitable channel code for the VP precoded system and its achievable sum-rate performance are not thoroughly investigated.

In general, the analysis of VP precoding sum-rate in conjunction with a channel code is difficult due to the following reasons: (i) γ is intractable in closed-form, (ii) the analytical complexity originating from channel and user data dependent v , and (iii) analytical intractability because of non-Gaussian noise (originate from modulo operation at user receivers) [13]. Under channel state information (CSI) error, one has to deal with additional challenge originated from the correlation between desired signal and multi-user interference that include both u and v . Previous studies [14]–[16] focus on sum-rate under perfect CSI and are based on the γ lower bound developed in [13] assuming a continuous uniform distributed input and/or based on the assumption of Gaussian “effective” noise or limited to conventional VP. On the other hand, the sum-rate under finite constellations such as m -QAM, non-Gaussian “effective” noise and practical channel coding can be significantly different.

To avoid aforementioned difficulties in the analysis and to design good channel codes for VP precoded transmission,

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we first develop a semi-analytic technique based on extrinsic information transfer (EXIT) charts [1], [17], [18]. Focusing on the coded transmission for VP precoded system, we then find the most suitable outer convolutional code (CC) under BICM-ID. Next, we investigate the achievable sum-rate of a fully loaded system where a BS with M antennas serves K single-antenna users with $M = K$. We compare the SNR necessary to reach a sufficiently low BER against the DPC limit for a given number of users K , a finite constellation modulation and the proposed channel code. As BER simulations are time consuming, and the computational complexity increases with higher K , the error performance behavior is also investigated through the developed EXIT chart based technique. Our investigation reveals that MMSE-VP precoding under the proposed channel code achieves a linear growth in sum-rate over K at a much lower SNR compared to conventional VP when perfect CSI available at the BS. With imperfect CSI knowledge due to limited feedback, we show that a linear growth in feedback bits (per user) is necessary to maintain the sum-rate linear growth. For ease of reference, a list of contributions is given below:

- With perfect CSI feedback, the most suitable CCs under BICM-ID for VP precoded transmissions are found by using an EXIT chart based technique. The sum-rate performance investigations of fully loaded systems (i.e., $M = K$) show that the proposed approach can achieve a linear sum-rate growth with the number of single-antenna users.
- To preserve the linear sum-rate growth of the proposed system under quantized CSI feedback, at least, a linear increase in the number of feedback bits per user is necessary.

II. SYSTEM MODEL

By using a suitable channel encoder which will be determined later, the i^{th} user information bit stream is first encoded to produce a transmission frame. Each group of m_c number of coded bits of i^{th} user are mapped into a constellation symbol u_i such as QPSK or QAM, of size 2^{m_c} using an appropriate mapping rule Ξ . In a given symbol period, the VP precoding is performed on the user data symbol vector \mathbf{u} for K users where $\mathbf{u} = [u_1, u_2, \dots, u_K]^T \in \mathbb{C}^{K \times 1}$. Particularly, consider symbol vector $\mathbf{d} = [d_1, d_2, \dots, d_K]^T \in \mathbb{C}^{K \times 1}$ constructed from \mathbf{u} and perturbation vector $\mathbf{v} = [v_1, v_2, \dots, v_K]^T \in \mathbb{C}^{K \times 1}$:

$$\mathbf{d} = \mathbf{u} + \tau \mathbf{v} \quad (1)$$

where τ is a positive scalar. \mathbf{v} is chosen to be a complex Gaussian integer vector, i.e., the elements of \mathbf{v} are in the set $\{a + jb | a, b \in \mathbb{Z}\}$. \mathbf{d} is precoded by the precoder matrix $\mathbf{W} \in \mathbb{C}^{M \times K}$ to produce precoded symbol vector $\mathbf{x} = [x_1, x_2, \dots, x_M]^T \in \mathbb{C}^{M \times 1}$, i.e.,

$$\mathbf{x} = \gamma^{-1} \mathbf{W} \mathbf{d} \quad (2)$$

where scaling factor $\gamma (> 0)$ is chosen to satisfy the BS power constraint [7], [10]–[12]. Each single-antenna user, say the i^{th} user, receives the signal $y_i \in \mathbb{C}$ over the channel $\mathbf{h}_i \in \mathbb{C}^{M \times 1}$ contaminated by the additive white Gaussian

noise (AWGN) $n_i \in \mathbb{C}$ and $n_i \sim \mathcal{CN}(0, N_0)$. We consider spatially uncorrelated Rayleigh fading environment, i.e., the elements of \mathbf{h}_i are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$.

In MMSE-VP, \mathbf{W} , \mathbf{v} , and γ are sequentially determined as follows [11], [19]:

$$\mathbf{W} = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H + K \rho \mathbf{I})^{-1}, \quad (3a)$$

$$\mathbf{v} = \arg \min_{\mathbf{v}} \|\mathbf{L}(\mathbf{u} + \tau \mathbf{v})\|^2, \quad (3b)$$

$$\gamma^2 = P^{-1} \mathbf{d}^H \mathbf{H} (\mathbf{H}^H \mathbf{H} + K \rho \mathbf{I})^{-2} \mathbf{H}^H \mathbf{d}. \quad (3c)$$

Here, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K]^H$, P is the BS instantaneous power constraint, i.e., $\|\mathbf{x}\|^2 \leq P$, $\rho = P/N_0$ is the SNR, and $\mathbf{L}^H \mathbf{L} = (\mathbf{H} \mathbf{H}^H + K \rho \mathbf{I})^{-1}$ found by the Cholesky decomposition. The integer-lattice least-square problem to find \mathbf{v} in (3b) can efficiently be solved via sphere encoder algorithm [7], [10], [11].

The received signal y_i at the i^{th} user can be modeled as

$$y_i = \gamma^{-1} (u_i + \tau v_i) + \hat{n}_i \quad (4)$$

where \hat{n}_i contains both receiver noise and interference introduced by the regularization coefficient $K \rho \mathbf{I}$ in \mathbf{W} . τ is chosen to provide symmetric decoding regions around constellation points and the choice $\tau = 2(|c|_{\max} + \Delta/2)$, where $|c|_{\max}$ is the absolute value of the largest magnitude constellation symbol and Δ is the spacing between constellation points, serves this purpose [7]. With this choice for τ , each user is able to apply the modulo function $f_{\tau/\gamma}(\cdot)$ on y_i independently (i.e., without cooperation among users) and to eliminate the effect of unknown v_i where $f_{\tau}(a) \triangleq a - \lfloor (a + \tau/2)/\tau \rfloor \tau$. Here $\lfloor \cdot \rfloor$ denotes the largest integer less than or equal to its argument [7]. Because of the multi-user interference in \hat{n}_i and the application of modulo function $f_{\tau/\gamma}(\cdot)$, the resulting “effective” noise can no-longer be considered Gaussian. Thus, the likelihood of $\hat{y}_i = f_{\tau/\gamma}(y_i)$ is computed similar to [7]:

$$p(\hat{y}_i | u_i) = \sum_{m=-\infty}^{\infty} \frac{1}{\pi N_0} \exp \left(-\frac{1}{N_0} \left| y_i - \frac{(u_i - m\tau)}{\gamma} \right|^2 \right). \quad (5)$$

In computations, the infinite summation over m is truncated by a sum of few terms from both sides of $m = 0$. The bit-wise likelihood ratio is computed using (5) and the bit-mapping rule Ξ .

Since our comparisons are done with the ergodic sum-capacity, we allow \mathbf{H} to be chosen randomly with every use. This randomly chosen effect can be obtained on a smoothly varying channel by using an interleaver over a long block of many consecutive channel uses.

III. CODED TRANSMISSION OVER VP PRECODING

Although the sum-capacity of DPC grows linearly with K when $M = K$, the sum-rate of linear ZF precoding saturates to a constant even if the idealistic Gaussian codebook is used [6]. This motivates one to investigate the sum-rate of non-linear VP precoding. As aforementioned, the analysis of VP sum-rate is non-trivial, even for Gaussian or continuous uniform distributed inputs. Suitable channel coding for VP precoded

transmission and its achievable sum-rate performance is more realistic and can be significantly different from idealistic Gaussian codebook and uniform inputs assumptions. We therefore first find a good channel code for VP precoding and then estimate the achievable sum-rate as follows.

Given that all K users encode by using code-rate r encoders and encoded bits are mapped to a m_c -bit constellation symbols, the achievable sum-rate R_{VP} is given by

$$R_{VP} = m_c r K \text{ (b/s/Hz)}. \quad (6)$$

To find R_{VP} , the minimum SNR required to achieve a given small BER is compared against DPC limit. For a given channel code, performing BER simulations is time consuming, and the computational complexity increases with higher K . Alternatively, we predict the waterfall region of BER curves or so-called turbo pinch-off through EXIT charts [18] and verify them via BER simulations.

A. BICM-ID Transmission over VP Precoding

BICM-ID is a bandwidth efficient channel coding scheme that can achieve a close-capacity performance under fading and AWGN environments. Here, we find a good CC in BICM-ID transmission combined with VP precoding and show that the proposed scheme can achieve a linear growth in sum-rate which is the same as sum-capacity growth of DPC.

The proposed system is shown in Fig. 1. The information sequence of i^{th} user \underline{s}_i is first encoded using a CC into a coded sequence \underline{c}_i . Then \underline{c}_i is interleaved by a bit-interleaver Π to become the interleaved sequence \tilde{c}_i . Each group of m_c bits in \tilde{c}_i is mapped to u_i according to a mapping rule Ξ . As described in Section II, VP precoding is then performed. At the receiver, as depicted in Fig. 1 and described in Section II, y_i is demodulated and decoded in an iterative manner. The receiver consists of a soft-output demodulator that follows the maximum a posteriori probability algorithm and a soft-input soft-output (SISO) channel decoder that uses the maximum a posteriori probability algorithm in [20].

Let I_{A_1} denote the mutual information between the a priori log-likelihood ratio and the transmitted coded bit and I_{E_1} denote the mutual information between the extrinsic log-likelihood ratio and the transmitted coded bit at the input and output of the demodulator. The demodulator EXIT characteristic I_{E_1} is then obtained as a function of I_{A_1} and SNR. These mutual-information can be calculated efficiently by Monte-Carlo experiments [18]. Similarly, let I_{A_2} and I_{E_2} be the mutual information representing the a priori knowledge and the extrinsic information of the coded bits at the input and output of the SISO decoder. The decoder EXIT characteristic I_{E_2} is found as a function of I_{A_2} . Note that because of the presence of the interleaver Π , this value does not depend on SNR. After being de-interleaved by Π^{-1} , the extrinsic output of the detector is used as the a priori input to the decoder, that is, $I_{A_2} = I_{E_1}$. Furthermore, after being interleaved, the extrinsic information of the decoder becomes the a priori information to be provided to the demodulator, that is, $I_{A_1} = I_{E_2}$. The convergence behavior of the iterative processing can be studied using the demodulator and the decoder EXIT curves.

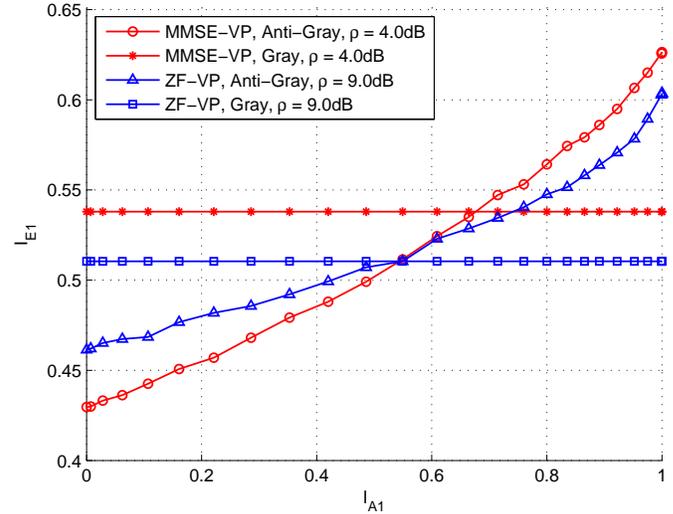


Fig. 2: Demodulator EXIT curves for BICM-ID transmission over MMSE-VP and conventional VP precoded systems with $r = 1/2$ CC and different mapping rules

In particular, both EXIT curves of the demodulator and the decoder are plotted in the same graph, but the axes of the decoder curve are swapped. When there is a tunnel opening between the EXIT curves, it allows for convergence of iterative decoding towards a low BER [18]. Using this EXIT chart technique, the most suitable CC can be found. Moreover, this technique is useful to study the turbo pinch-off by avoiding the time consuming BER Monte-Carlo simulations.

B. Code design using EXIT charts

In this subsection, we find a good channel code for proposed VP based system through EXIT chart matching. Fig. 2 shows EXIT curves of the demodulator for Gray and anti-Gray mapping rules for $r = 1/2$. One observes that the EXIT curve with Gray mapping is completely horizontal. This makes Gray mapping a good match with powerful turbo-like codes, because EXIT curves of these codes are also almost horizontal. On the other hand, EXIT curves of anti-Gray mappings have slopes, which make them suitable for other class of codes having a decayed EXIT curve such as CC.

Our objective is to find the most suitable $r = 1/2$ CC from the maximum-free-distance CCs reported in [21]. Fig. 3 shows the EXIT curves of demodulator and two selected CC decoders: i) [53, 75] and ii) [21675, 27123]. We observe that the EXIT curves of the [53, 75] CC decoder and demodulator quickly intersect and the intersection point falls in the lower left quadrant of the EXIT plane. On the other hand, there is a narrow tunnel opening between the EXIT curves allowing convergence of iterative decoding towards low BER for [21675, 27123] CC decoder and demodulator [18]. This result predicts that for [21675, 27123] CC, the turbo pinch-off happens for MMSE-VP and ZF-VP (i.e., conventional VP with ZF precoder) at SNRs $\rho = 4.0$ dB and $\rho = 9.0$ dB, respectively. As SNR increases, the tunnel opening widens and consequently, iterative decoding convergence faster leading to early termination and small BER levels [18]. To verify

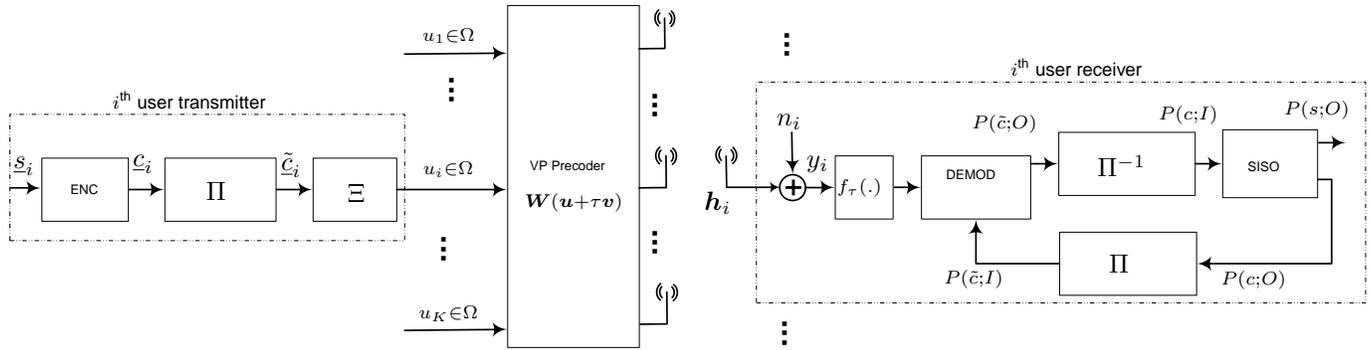


Fig. 1: System model: BICM-ID transmission over VP precoding

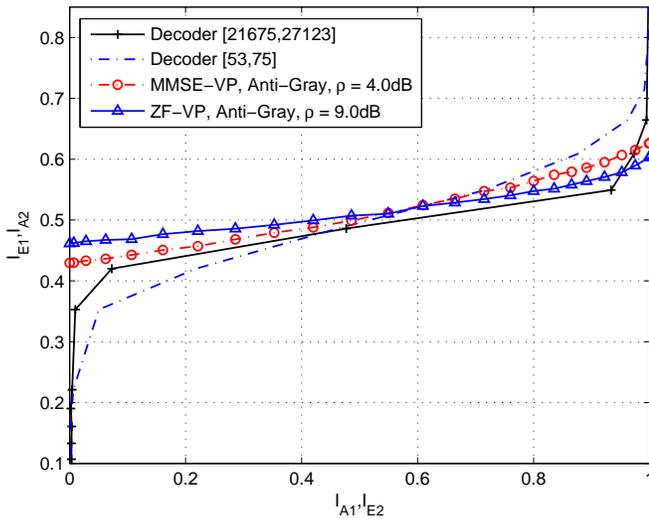


Fig. 3: EXIT chart of demodulator with $r = 1/2$ CC decoder for BICM-ID transmission over MMSE-VP and conventional VP precoded systems with $M = K = 4$

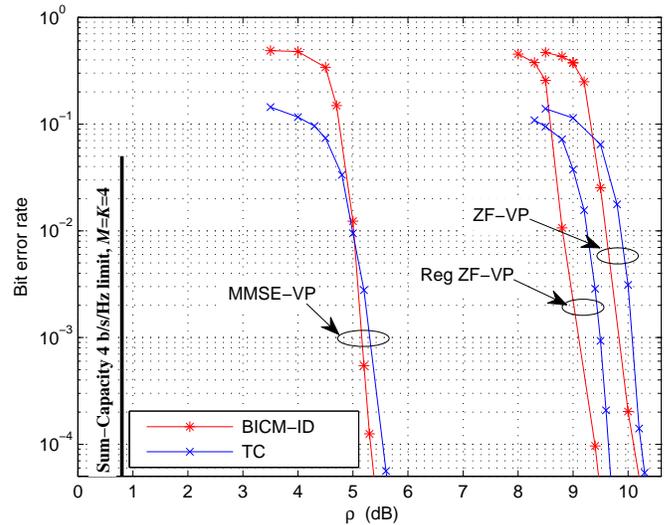


Fig. 4: BER performance of $r = 1/2$ BICM-ID and TC transmissions over MMSE-VP and conventional VP (Reg ZF-VP and ZF-VP) precoded systems with $M = K = 4$

this prediction, we plot the respective BER curves in Fig. 4 for 12000-bit random interleaver. We observe that the EXIT chart predicted turbo pinch-off SNRs match well with BER curves. For comparison, BER performance of rate-1/2 turbo code (TC) and the DPC limit of sum-capacity 4 b/s/Hz with $M = K = 4$ are depicted. Similar to [7], rate-1/2 TC is constructed from the rate-1/3 parent code with systematic component, feedforward polynomial $1 + D + D^3$, feedback polynomial $1 + D^2 + D^3$ and using the non-systematic bit puncturing pattern [10],[01]. The MMSE-VP $M = K = 4$ (i.e., $R_{VP} = 4$ b/s/Hz (cf. (6))) with BICM-ID achieves a BER= 10^{-4} about 4.5 dB SNR away from DPC limit and the SNR gains are significant compared to conventional VP with regularized-ZF (Reg ZF-VP) considered in [7]. Moreover, by considering EXIT chart convergence properties for different K , one can predict the turbo pinch-off to compare with the DPC limit without performing BER simulations.

IV. SUM-RATE PERFORMANCE

Here, by using the EXIT chart technique, the achievable rate of proposed system is compared against DPC limit and ZF

precoding with Gaussian codebook. It should be emphasized that the achievable sum-rate of the proposed VP precoding based system is realistic because of the use of practical channel coding with finite constellation modulation.

We first study the convergence behavior of iterative processing through EXIT charts and predict the turbo pinch-off region. Fig. 5 depicts demodulator EXIT curves at $\rho = 4.0$ dB for varying K along with $r = 1/2$ CC [21675, 27123] decoder EXIT curve. The results in Fig. 5 show that demodulator EXIT curves almost overlap for different K and match well with decoder EXIT curve. Therefore, the turbo pinch-off for all K values happens approximately at $\rho = 4.0$ dB. To verify this prediction, Fig. 6 plots the BER simulations for various K values. We observe that BER curves in Fig. 6 almost coincide, which confirms the prediction from Fig. 5. We have a 4-QAM ($m_c = 2$) system along with the rate 1/2 CC ($r = 1/2$) and therefore, from (6), $R_{VP} = K$ b/s/Hz. To compare, we plot the DPC sum-capacity over K at $\rho = 4.0$ dB along with ZF precoding (Gaussian codebook) in Fig. 7. Observe that, due to the ill-conditioned channel, the sum-rate of ZF precoding

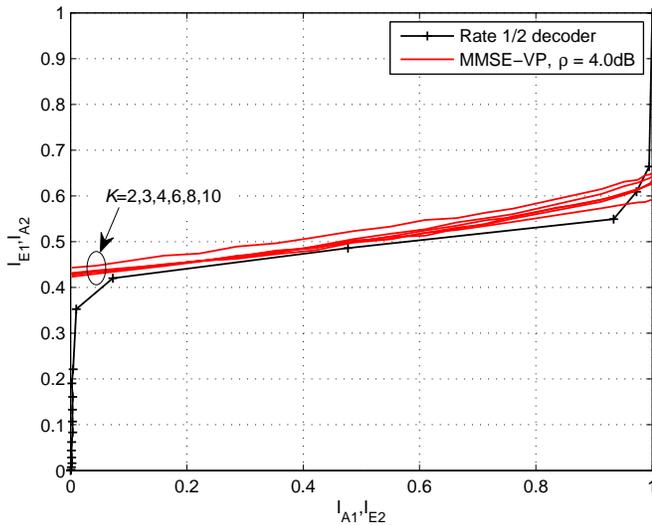


Fig. 5: EXIT chart of demodulator and decoder for different number of serving users K

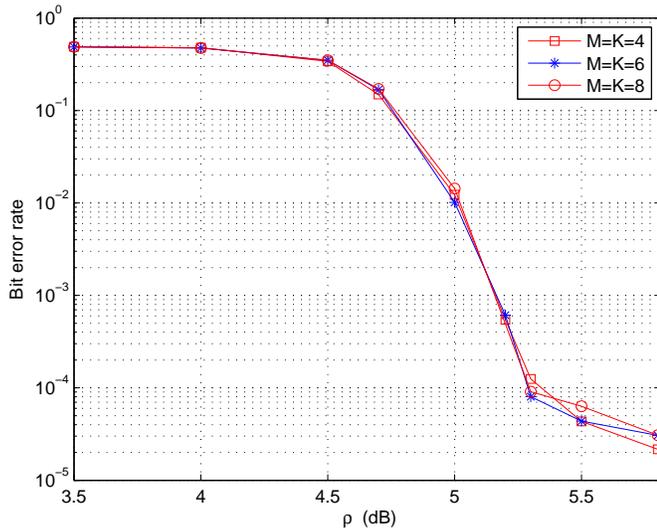


Fig. 6: BER performance of $r = 1/2$ BICM-ID transmission over MMSE-VP precoded system for different number of serving users K with $M = K$

saturates to a constant at large values of K when $M = K$ [6]. This saturation at an asymptotically high K and M , i.e., $K \rightarrow \infty$ and $M \rightarrow \infty$ in the fixed ratio $M/K = 1$, can be approximated by $C_{zf} = \rho \log_2(e)$ b/s/Hz [6] and depicted in Fig. 7 for comparison. We have also investigated $r = 1/3$ system and the turbo pinch-off at $\rho = 0.7$ dB is observed through both EXIT charts and BER curves similar to Figs. 5 and 6. As a result, a linear growth of $R_{VP} = 2K/3$ b/s/Hz is achieved. We have also performed the analysis for BICM-ID with the $r = 1/2$ CC in conjunction with ZF-VP precoding and compared with the bounds presented in [13], [16]. The results indicate a linear growth in sum-rate at $\rho = 9.0$ dB. These results show that the VP precoding achieves a linear growth of sum-rate under practical channel coding with finite constellation which is the same as DPC transmission.

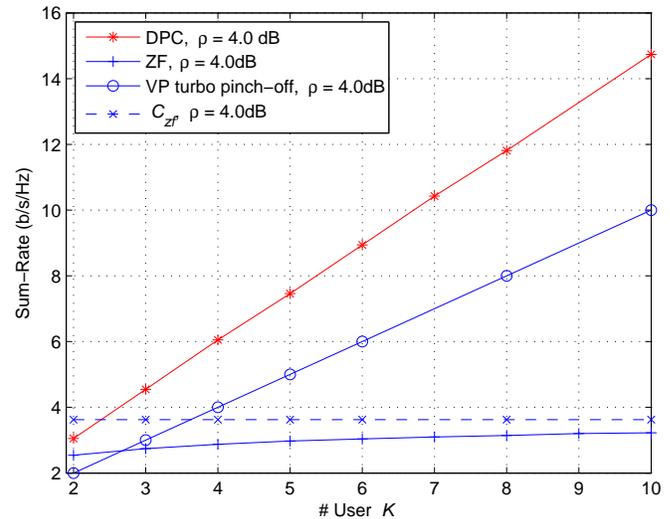


Fig. 7: Sum-rate performance of BICM-ID transmission over MMSE-VP precoded system for different number of serving users K with $M = K$

A. Impact of CSI Error on the Performance

In practice, it is difficult to obtain perfect knowledge about \mathbf{H} due to errors. We model

$$\mathbf{H} = \hat{\mathbf{H}} + \Delta\mathbf{H} \quad (7)$$

where the base-station knows only $\hat{\mathbf{H}}$ and the elements in channel error matrix $\Delta\mathbf{H}$ are i.i.d. $\mathcal{CN}(0, \sigma_e^2)$. Note that $\sigma_e^2 = 0$ corresponds to perfect CSI scenario. When this model represents CSI quantization, a rate-distortion theory based codebook of size 2^B is used where $B = M \log_2(1/\sigma_e^2)$. Thus, per user feedback overhead is B bits.

From the developed EXIT chart based analysis, the VP sum-rate under CSI error can also be investigated. The MMSE-VP precoder $\hat{\mathbf{W}} = \hat{\mathbf{H}}^H [\hat{\mathbf{H}} \hat{\mathbf{H}}^H + K(\rho + \sigma_e^2) \mathbf{I}]^{-1}$ which can effectively tolerate CSI errors is used [8], [19]. Using the model given in (7), we have investigated the convergence properties through EXIT charts and BER performance for the proposed BICM-ID with the $r = 1/2$ CC and MMSE-VP. At a given σ_e^2 , we observe in simulations that EXIT curves overlap almost at the same SNR value. Therefore, turbo pinch-off under CSI error also happens at the same SNR. These results are similar to the ones illustrated in Figs. 5 and 7 for the case of perfect CSI, but at a higher pinch-off SNR. The turbo pinch-off SNR for $\sigma_e^2 = 0, 2^{-6}, 2^{-5}, 2^{-4}, 2^{-3}$ values happen respectively at 4.0, 4.3, 4.7, 5.5, 6.5 dB SNR levels. Clearly, the more imperfect the CSI is, the higher the pinch-off SNR is to compensate for this imperfection. Similar to the case of perfect CSI in Fig. 7, a linear growth in sum-rate over K is achievable for imperfect CSI as long as σ_e^2 is invariant to K . As we consider fully loaded system with $M = K$ and the feedback load per user is $B = M \log_2(1/\sigma_e^2)$, a linear growth in feedback overhead (per user) is required to maintain a linear growth in sum-rate under the imperfect CSI.

V. CONCLUSION

We investigate a suitable BICM-ID transmission for VP precoded system in a fully loaded multi-user downlink with $M = K$. First, by developing a semi-analytic EXIT chart based technique, the most suitable outer CC under BICM-ID is found for VP precoded transmission. We then investigate the sum-rate performance of the proposed channel code for VP precoded system. It is shown that, over the number of serving users, the BICM-ID based VP precoded transmission can achieve the same linear growth of capacity achieving DPC scheme but at a much lower SNR compared to conventional VP. Under limited CSI feedback, the results indicate that a linear growth in the feedback load per user is necessary to maintain the linear growth of the sum-rate.

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