Optimization in Wireless Multi-relay Networks

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May 13, 2009

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Outline

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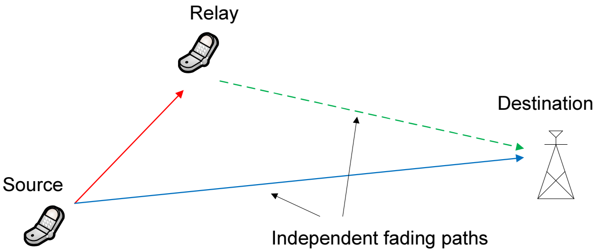
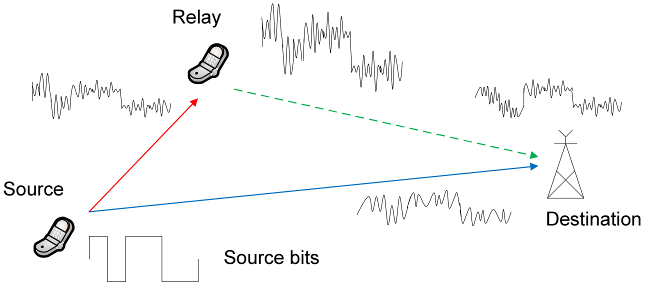
Cooperative Communication - Overview



- A new form of spatial diversity.
- Users cooperate to relay signals of each other, and emulate a virtual array of transmit antennas.
- Huge potential in improving the reliability of the wireless network.

Amplify-and-forward - Two stages of transmission



- First stage: Source (S) transmits to both Relay (R) and Destination (D).
- Second stage: Relay amplifies the received signal and forwards it to Destination.
- Destination combines the two received signals to decode.

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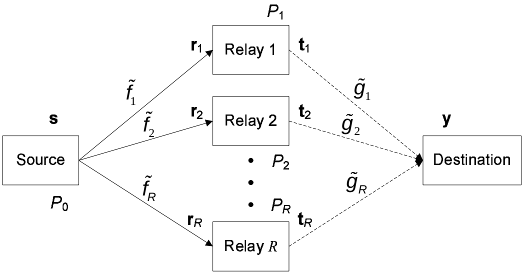
System Model (in Part I)



- One antenna per node, used for both TX and RX.
- R relays work in half-duplex mode, Amplify-and-Forward (AF) protocol is considered.
- No direct link from source to destination.

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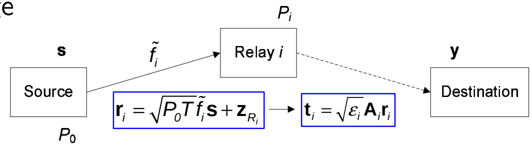
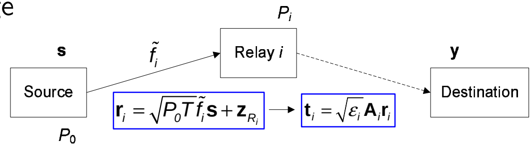
Distributed Space-Time Coding (DSTC)

- First stage

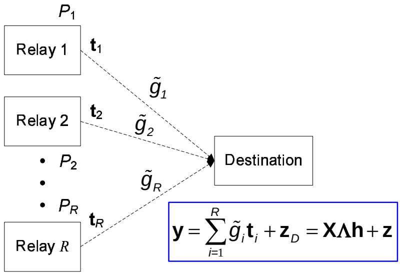


Distributed Space-Time Coding (DSTC)

- First stage



- Second stage



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Distributed Space-Time Coding (DSTC) (cont.)

- The mathematical model is

$$\mathbf{y} = \sum_{i=1}^{R} \tilde{g}_i \mathbf{t}_i + \mathbf{z}_D = \mathbf{X}\boldsymbol{\Lambda}\mathbf{h} + \mathbf{z}$$

where

$$\begin{aligned}
\mathbf{X} &= [\mathbf{A}_1\mathbf{s}, \ldots, \mathbf{A}_R\mathbf{s}] \\
\boldsymbol{\Lambda} &= \mathrm{diag}\left(\sqrt{\varepsilon_1 \sigma_{F_1}^2 \sigma_{G_1}^2}, \ldots, \sqrt{\varepsilon_R \sigma_{F_R}^2 \sigma_{G_R}^2}\right) \\
\mathbf{h} &= \left[f_1^{(*)} g_1, \ldots, f_R^{(*)} g_R\right]^{\mathcal{T}} \\
\mathbf{z} &= \frac{1}{\sqrt{P_0 T \sigma_R^2}} \sum_{i=1}^{R} \sqrt{\varepsilon_i \sigma_{G_i}^2} g_i \mathbf{A}_i \mathbf{z}_{R_i}^{(*)} + \frac{1}{\sqrt{P_0 T \sigma_D^2}} \mathbf{z}_D.
\end{aligned}$$

- $\mathbf{X}$ is now a distributed space-time codeword.

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Early Works and My Works in DSTC

- Applying ST coding in MIMO systems to relay networks.
- Coherent networks *(CSI of $S \rightarrow R$, $R \rightarrow D$ known):* linear dispersion DSTC [Jing06], orthogonal DSTC [Jing07].
- Partially coherent networks *(only CSI of $R \rightarrow D$ known):* differential DSTC [Kiran07].
- Noncoherent networks *(CSI unknown):* cyclic DSTC [Oggier06].
- My approach:
  - Propose Fourier-based Distributed Unitary Space-Time Modulation (DUSTM): design source signal $\mathbf{s}_k$ and relaying matrix $\mathbf{A}_i \longrightarrow$ design $\mathbf{X}_k$.
  - Provide a unified analysis for partially-coherent and noncoherent networks.

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DUSTM in a Partially Coherent Network

- The ML decoding is to maximize the probability $p(\mathbf{y}|\mathbf{X}_k, \{g_i\})$. The decoding rule could be found as

$$\mathbf{X}_{ML} = \arg \max_{\mathbf{X}_k=\mathbf{X}_1,\ldots,\mathbf{X}_L} \mathbf{y}^{\mathcal{H}}\mathbf{X}_k\mathbf{C}\mathbf{X}_k^{\mathcal{H}}\mathbf{y}, \qquad (1)$$

where

$$\mathbf{C} = \text{diag}\left(\frac{\beta_1|g_1|^2}{\gamma+\beta_1|g_1|^2},\ldots,\frac{\beta_R|g_R|^2}{\gamma+\beta_R|g_R|^2}\right)$$

$$\gamma = \frac{1}{P_0 T}\left(1+\sum_{i=1}^{R}\varepsilon_i\sigma_{G_i}^2|g_i|^2\right)$$

$$\beta_i = \varepsilon_i\sigma_{F_i}^2\sigma_{G_i}^2.$$

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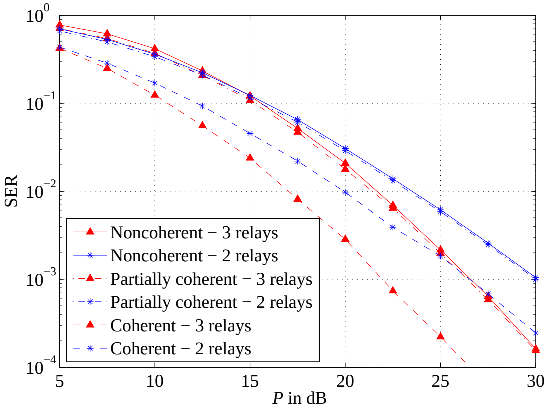
DUSTM in a Noncoherent Network

- $p(\mathbf{y}|\mathbf{X}_k)$ does not appear to have a closed-form expression. The optimal ML decoder is *unavailable*.
- A suboptimal Generalized Likelihood Ratio Test (GLRT) decoder can be derived as

$$\mathbf{X}_{GRTL} = \arg \max_{\mathbf{X}_k = \mathbf{X}_1, \ldots, \mathbf{X}_L} \mathbf{y}^{\mathcal{H}} \mathbf{X}_k \mathbf{X}_k^{\mathcal{H}} \mathbf{y}. \tag{2}$$

- *Remarks:* The difference between the ML decoder in (1) and the GLRT decoder in (2) is the existence of the matrix $\mathbf{C}$, which contains the CSI of the $R \rightarrow D$ channels.

Partially-coherent vs. Noncoherent



- Symbol error performance of DUSTM: $\sigma_{F_i}^2 = 10$ and $\sigma_{G_i}^2 = 1$.

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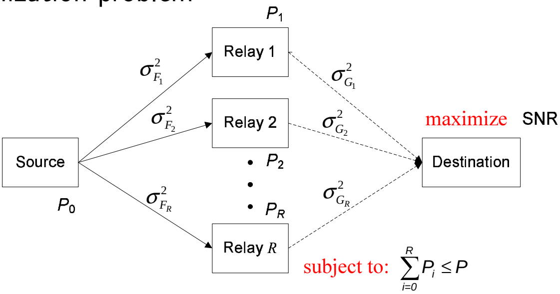
The Optimization Problem

- The average SNR at the destination:

$$\mathrm{SNR} = \frac{P_0 T}{R N_0} \frac{\sum_{i=1}^{R} \varepsilon_i \sigma_{F_i}^2 \sigma_{G_i}^2}{1 + \sum_{i=1}^{R} \varepsilon_i \sigma_{G_i}^2}. \quad (3)$$

- The optimization problem



Balanced Networks

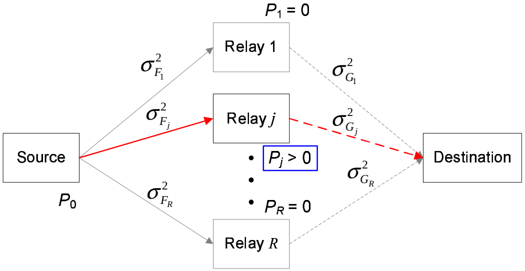
- Early works *only* consider the special case: $\sigma_{F_i}^2 = \sigma_{G_i}^2 = 1$, the "Equal PA" scheme $P_0 = P/2$, $P_i = P/(2R)$ is optimal.
- My approach: Study the PA scheme for arbitrary $\sigma_{F_i}^2$ and $\sigma_{G_i}^2$.
- Balanced networks: $\sigma_{F_1}^2 = \ldots = \sigma_{F_R}^2 = \sigma_F^2$, and $\sigma_{G_1}^2 = \ldots = \sigma_{G_R}^2 = \sigma_G^2$, the optimal PA scheme is

$$P_0 = \begin{cases} \frac{\sqrt{(P\sigma_F^2+N_0)(P\sigma_G^2+N_0)}-(P\sigma_G^2+N_0)}{\sigma_F^2-\sigma_G^2}, & \text{if } \sigma_F^2 \neq \sigma_G^2 \\ P/2, & \text{if } \sigma_F^2 = \sigma_G^2 \end{cases}$$

$$P_1 = \ldots = P_R = (P - P_0)/R.$$

Unbalanced Networks

- The network is called unbalanced if the conditions $\sigma^2_{F_1} = \ldots = \sigma^2_{F_R}$ and $\sigma^2_{G_1} = \ldots = \sigma^2_{G_R}$ are not met.
- To get the maximum SNR, the relay power is allocated to the *best* relay, say the *j*th relay. Thus, only one fading path is active $\rightarrow$ compromise the performance of the DSTC.



Balancing the Unbalanced Networks

- The amount of fading (AoF) is a common measure of fading severity in a fading channel model.
- Establish the condition to minimize the amount of fading in a relay network to balance the fading statistics of each $S \rightarrow R \rightarrow D$ link.
- With the AoF constraint, the optimal PA scheme

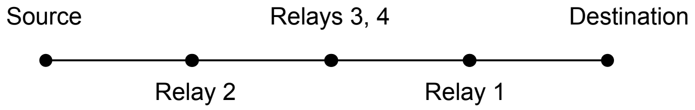
$$P_0 = \begin{cases} \frac{\sqrt{(Pa+c)(Pb+c)}-(Pa+c)}{b-a}, & \text{if } b \neq a \\ P/2, & \text{if } b = a \end{cases}$$

$$P_i = \frac{P - P_0}{P_0 b + c} \cdot \frac{P_0 \sigma^2_{F_i} + N_0}{\sigma^2_{F_i} \sigma^2_{G_i}}, \qquad i = 1, \ldots, R, \qquad (4)$$

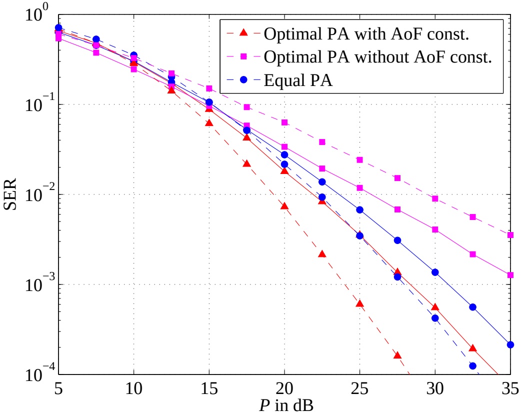
where a, b, and c are parameters.

Some Remarks

- The proposed PA scheme achieves the maximum diversity order in all coherent, partially coherent, and noncoherent relay networks.
- The proposed PA scheme yields a significant performance advantage over the "equal PA" scheme.
- Consider the unbalanced network:

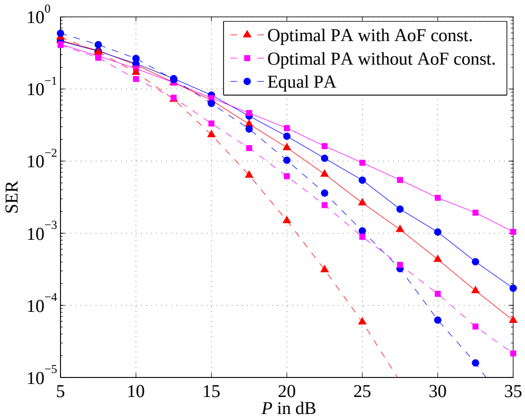


Noncoherent Network



- Distributed Unitary Space-Time Modulation (USTM) is applied to the noncoherent network with 2 and 3 relays.

Coherent Network



- Distributed Orthogonal Space-Time Block Coding (OSTBC) is applied to the coherent network with 2 and 4 relays.

Optimal Training in DSTC

- Coherent ML decoding of DSTC requires the knowledge of channel coefficient vector $\mathbf{h}$.
- Need to estimate $\mathbf{h}$ at the destination:
  - Send a known training sequence $\mathbf{u}$ from the source

$$\mathbf{y}_T = \mathbf{X}_T \mathbf{\Lambda} \mathbf{h} + \mathbf{z}_T,$$

  where $\mathbf{X}_T = [\mathbf{A}_1 \mathbf{u}^{(*)}, \ldots, \mathbf{A}_R \mathbf{u}^{(*)}]$.
  - Estimate $\mathbf{h}$ from $\mathbf{y}_T$, $\mathbf{X}_T$, and $\mathbf{\Lambda}$.
- Early works studies the optimal design of $\mathbf{X}_T$ [Gao08].
- My approach: find the optimal PA scheme in training phase, and investigate the impact of imperfect CSI to the coherent code.

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Mean-Square Error in Channel Estimation

- Maximum Likelihood (ML) estimation:

$$\mathrm{cov}\left(\boldsymbol{\Delta}_h\right) = \bar{\gamma}\boldsymbol{\Lambda}^{-1}(\mathbf{X}_T^{\mathcal{H}}\mathbf{X}_T)^{-1}\boldsymbol{\Lambda}^{-1}.$$

where $\bar{\gamma} = \mathbb{E}[\gamma] = \frac{1}{P_0 T}\left(1 + \sum_{i=1}^{R} \varepsilon_i \sigma_{G_i}^2 |g_i|^2\right)$

- Minimum Mean-Square Error (MMSE) estimation

$$\mathrm{cov}(\boldsymbol{\Delta}_h) = \left(\mathbf{I}_R + \frac{1}{\bar{\gamma}}\boldsymbol{\Lambda}\mathbf{X}_T^{\mathcal{H}}\mathbf{X}_T\boldsymbol{\Lambda}\right)^{-1}.$$

- The mean-square error (MSE) is minimized when
  1. $\mathbf{X}_T$ is orthogonal [Gao08].
  2. The optimal PA scheme with the minimum amount of fading constraint.

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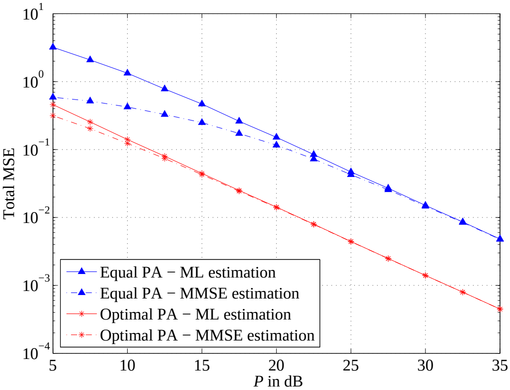
Coherent Network



- Total MSE achieved with ML and MMSE estimators, and with the optimal and equal PA schemes.

Mismatched Decoding

- Recall the system model

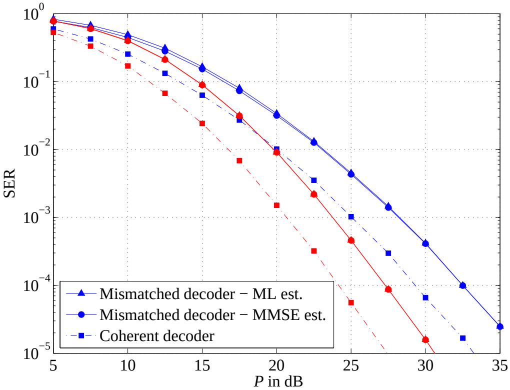
$$\mathbf{y} = \mathbf{X}\boldsymbol{\Lambda}\mathbf{h} + \mathbf{z}.$$

- Use the estimated CSI $\hat{\mathbf{h}}$

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}_k} \left\| \mathbf{y} - \mathbf{X}_k \boldsymbol{\Lambda} \hat{\mathbf{h}} \right\|^2.$$

- The same diversity order is achieved with imperfect CSI estimation as with perfect CSI.

Coherent Network



- Error performance of DSTC with different types of decoding. Red lines: Optimal PA, Blue lines: Equal PA.

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Introduction to Part II

- With full CSI, the relays can beam the retransmitted signal to the destination ⇒ received signal is coherently constructed.
- Early works *only* consider power allocation at the relays for a one-source one-destination network.
- My approach: Find optimal power allocation for a multiple-source multiple-destination network
  - (i) Minimizing the sum relay power with guaranteed quality of service (QoS).
  - (ii) Maximizing the joint SNR margin subject to power constraints at the relays.
  - Apply convex optimization to investigate the problems.
  - Propose simple and fast converging algorithms.

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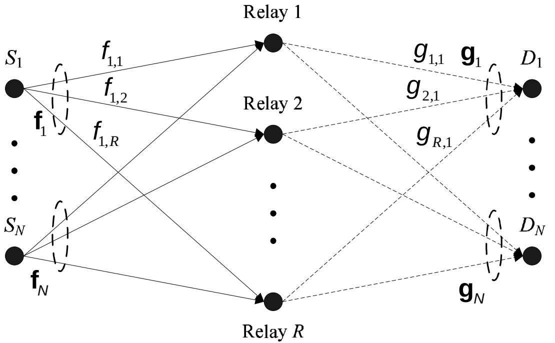
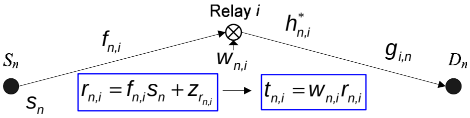
System Model (in Part II)



- N users ($S_n - D_n$) compete for the power resource at the R relays.

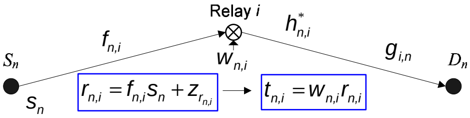
System Model

- First stage

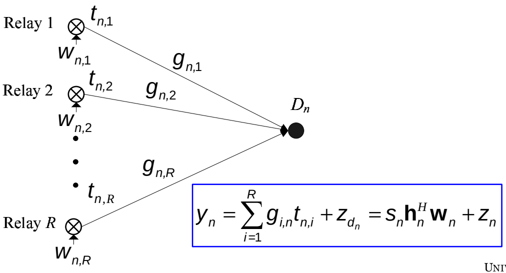


System Model

- First stage



- Second stage



Instantaneous SNR

- The received signal at the nth destination is:

$$y_n = \mathbf{g}_n^{\mathcal{T}} \mathbf{t}_n + z_{d_n} = \mathbf{h}_n^{\mathcal{H}} \mathbf{w}_n s_n + z_n.$$

- Instantaneous SNR at destination-n

$$\mathrm{SNR}_n = \frac{\sigma_{S_n}^2 |\mathbf{h}_n^{\mathcal{H}} \mathbf{w}_n|^2}{\sigma_R^2 \|\mathbf{G}_n^{1/2} \mathbf{w}_n\|^2 + \sigma_D^2}.$$

- Let p_n be the total relay power allocated for user-n

$$p_n = \mathbb{E}\left[\|\mathbf{t}_n\|^2\right] = \mathbf{w}_n^{\mathcal{H}} \mathbf{D}_n \mathbf{w}_n.$$

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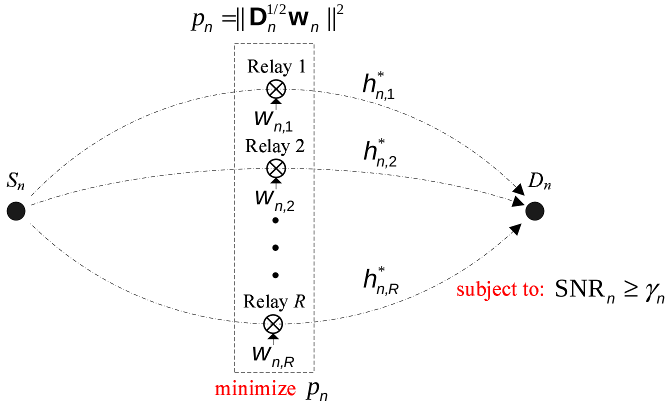
Without Per-Relay Power Constraints

- Minimize the sum relay power with guaranteed QoS.
- Can be performed separately for each user



Without Per-Relay Power Constraints - Solutions

- Second-order cone programming (SOCP).
- Find p_n directly, determine the optimal beamformer $\mathbf{w}_n$ accordingly.

$$\mathcal{P}_n(\gamma_n) = \begin{cases} \underset{p_n}{\text{minimize}} & p_n \\ \text{subject to} & \sum_{i=1}^{R} \frac{a_{n,i} p_n}{b_{n,i}+p_n} \geq \gamma_n, \end{cases}$$

where $a_{n,i}$ and $b_{n,i}$ are parameters.

- Simple fixed point iteration

$$p_n^{(t+1)} = \frac{\gamma_n}{\sum_{i=1}^{R} \frac{a_{n,i}}{b_{n,i}+p_n^{(t)}}} \triangleq f_n(p_n^{(t)}).$$

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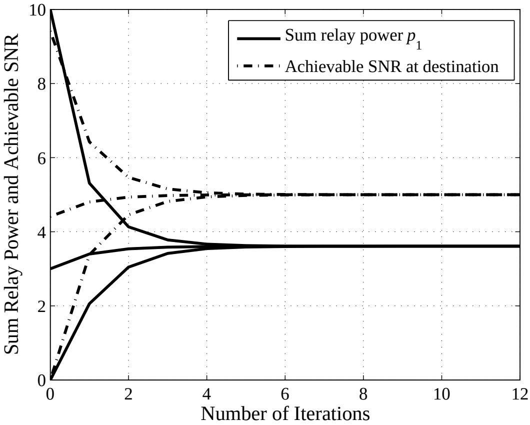
Convergence

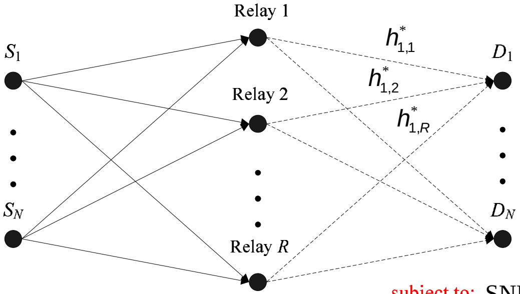


- Fixed point iteration with different starting points.

With Per-Relay Power Constraints

- Power constraint at each relay.
- Uniformly minimize the margin $P_i/P_i^{\max}$, denoted as α.

$$P_i = \sum_{n=1}^{N} \| \mathbf{D}_n^{1/2} \mathbf{E}_i \mathbf{w}_n \|^2$$



minimize $\alpha \sum_{i=1}^{R} P_i^{\max}$

subject to: $\text{SNR}_n \geq \gamma_n$

$P_i \leq \alpha P_i^{\max}$

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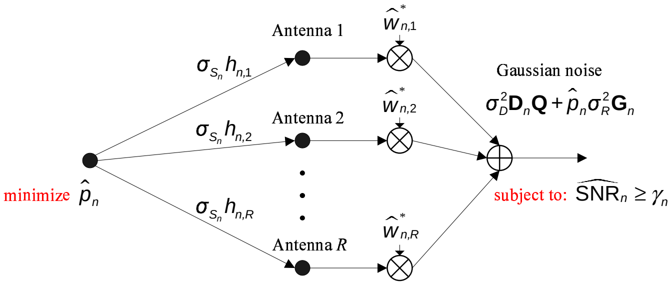
Without Per-Relay Power Constraints - Solutions

- Second-order cone programming (SOCP).
- Study to dual problem:
  - The Lagrangian, the dual function, and the dual problem.
  - An equivalent virtual uplink channel to the dual problem



  - Solve the dual problem, and find the optimal beamfomer $\mathbf{w}_n$.

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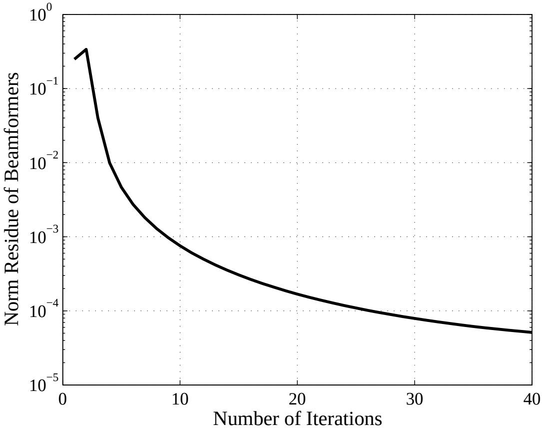
Convergence



- Convergence through the plot of $\sum_{n=1}^{N} \|\mathbf{w}_n - \mathbf{w}_n^\star\|$.

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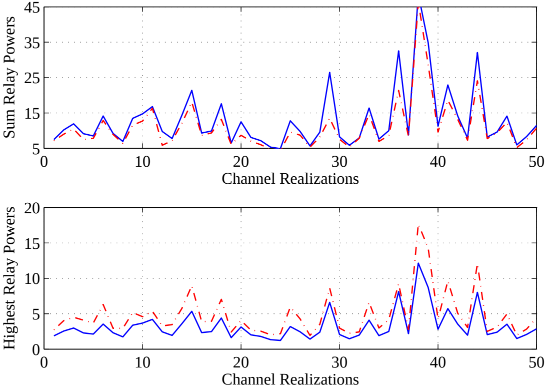
Power Consumption Comparison



- Red lines: without per-relay power constraints vs. Blue lines: with per-relay power constraints.

Sum Relay Power Constraint

- Jointly maximize the SNR-margin

$$\begin{aligned} \underset{\mathbf{w}_1,\ldots,\mathbf{w}_N}{\text{maximize}} \quad & \min_n \frac{\mathrm{SNR}_n}{\gamma_n} \\ \text{subject to} \quad & P_{\mathrm{relay}} \leq P_{\mathrm{relay}}^{\max}. \end{aligned}$$

- Solutions: bisection method, modified fixed-point iteration to directly find the optimal solution:

$$\tilde{p}_n = \frac{\gamma_n}{\sum_{i=1}^{R} \frac{a_{n,i}}{b_{n,i} + p_n^{(t)}}},$$

then normalize the result

$$p_n^{(t+1)} = \frac{P_{\mathrm{relay}}^{\max}}{\sum_{l=1}^{N} \tilde{p}_l}.$$

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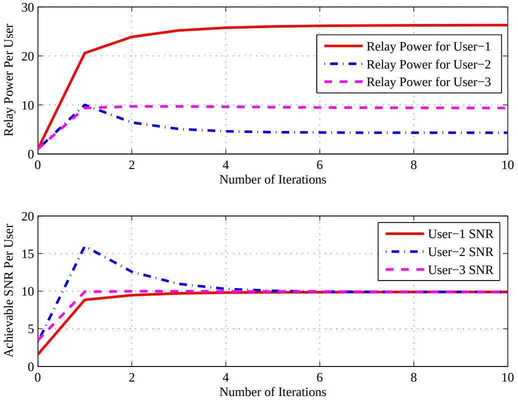
Convergence



- Convergence of the modified fixed point iteration for each user and the corresponding SNR.

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Sum Relay Power Constraint

- Jointly maximize the SNR-margin

$$\begin{aligned} \underset{\mathbf{w}_1,\ldots,\mathbf{w}_N}{\text{maximize}} \quad & \min_n \frac{\mathrm{SNR}_n}{\gamma_n} \\ \text{subject to} \quad & P_i \leq P_i^{\max}, \; i = 1, \ldots, R. \end{aligned}$$

- Solutions: bisection method, an iterative algorithm to directly find the optimal solution.

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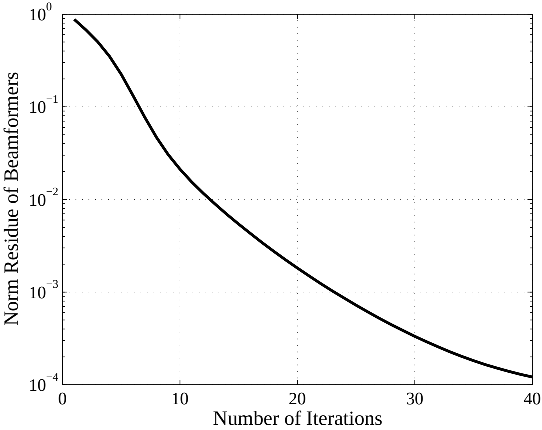
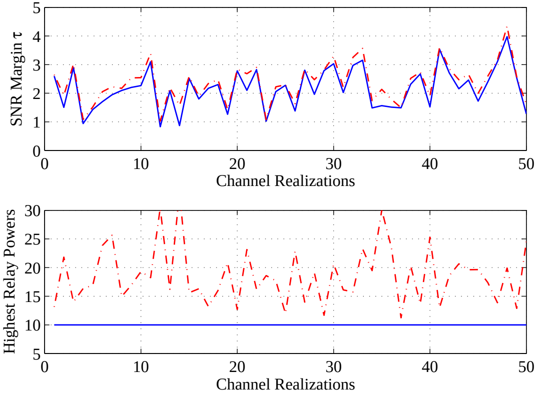
Convergence



- Convergence through the plot of $\sum_{n=1}^{N} \|\mathbf{w}_n - \mathbf{w}_n^\star\|$.

SNR & Power Consumption Comparison



- Red lines: sum relay power constraint
  vs. Blue lines: per-relay power constraints.

Summary

- Distributed space-time coding: code design, power allocation, training and mismatched decoding.
- Distributed beamforming for multi-source multi-destination: power minimization with guaranteed QoS at the destinations, SNR margin maximization with power constraints at the relays.

QUESTIONS?

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Contributions

1. "Power Allocation and Error Performance of Distributed Unitary Space-Time Modulation in Wireless Relay Networks", *to appear in IEEE Trans. on Veh. Tech.*
2. "Channel Estimation and Performance of Mismatched Decoding in Wireless Relay Networks", *submitted to IEEE Trans. on Wireless Comm.*
3. "Resource Allocation in Wireless Multiuser Multi-relay Networks", *in preparation.*
4. "Distributed Beamforming in Relay-Assisted Multiuser Communications", *in Proc. IEEE ICC' 09*, Jun. 2009.
5. "A Novel Power Allocation Scheme for Distributed Space-Time Coding", *in Proc. IEEE ICC' 09*, Jun. 2009.
6. "Channel Estimation and Performance of Mismatched Decoding in Wireless Relay Networks", *in Proc. IEEE ICC' 09*, Jun. 2009.
7. "Power Allocation and Distributed Beamforming Optimization in Relay-Assisted Multiuser Communications", *in Proc. IWCMC' 09*, Jun. 2009.
8. "Distributed Unitary Space-Time Modulation in Partially Coherent and Noncoherent Relay Networks", *in Proc. IPSPCS' 08*, Dec. 2008.
9. "Distributed Beamforming in Multiuser Multi-relay Networks with Guaranteed QoS", *submitted to Globecom' 09.*
10. "SNR Maximization and Distributed Beamforming in Multiuser Multi-relay Networks", *submmited to Globecom' 09.*

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