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Joint beamforming design and base-station assignment in a coordinated multicell system

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Abstract: This study is concerned with the downlink beamforming designs in a coordinated multicell system with dynamic base-station (BS) assignment. At each cell, a multiple-antenna BS employs linear beamforming to send multiple data streams to its assigned mobile-stations (MSs). Exploiting multicell coordination, the multiple BSs jointly optimise the beamformers and the BS-MS assignments to enhance the overall system performance. With per-BS power constraints, considered are the coordinated beamforming problems under the following two design criteria: (i) minimising the transmit power margin at the BS with a set of target signal-to-interference-plus-noise ratios (SINR) at the MSs and (ii) jointly maximising the minimum SINR margin at the MSs. As the original problem formulations are shown to be non-convex integer programs, which are combinatorially hard, the authors propose an efficient convex relaxation approach to solve the problems with low complexity. Simulations show that the convex relaxation-based assignment schemes significantly outperform heuristic fixed assignment schemes.

1 Introduction

Since the spectrum resource is limited, deploying frequency-reuse in wireless communications is inevitable to support the increasing number of wireless terminals and the increasing demand for higher transmission rates of next-generation wireless networks. However, because of the broadcast nature of the wireless medium, frequency-reuse may lead to the problem of inter-cell interference (ICI), that is, co-channel interference. In the latest 3rd Generation Partnership Project (3GPP) long-term evolution (LTE)-Advanced Release 10, coordinated multi-point (CoMP) transmission/reception has been proposed as an enabling technique to improve the system's coverage, throughput and efficiency [1]. With CoMP, the multicell network actively deals with the ICI and even takes advantage of the inter-cell transmissions to better the system performance. In the downlink channel, CoMP coordinates the simultaneous information transmissions from multiple base-stations (BSs) to the MSs, especially the ones in the cell-edge region. In this work, it is our interest to investigate this transmission/reception paradigm and examine efficient algorithms to realise its performance advantages.

Under the downlink CoMP architecture, two different modes are currently under consideration: 'multiple-input multiple-output (MIMO) cooperation' and 'interference coordination', depending on the level of coordination among the cells. In the MIMO cooperation mode, the antennas from the multiple BSs form a large single antenna array [2, 3]. Data streams intended for all the mobile stations (MSs) are jointly processed and transmitted from all the antennas. Apparently, this approach is the most complex CoMP mode (highest level of coordination) as it requires a significant amount of signalling among the BSs.

In a lower level of coordination, a coordinated multicell system may allow a BS to transmit the data only to the MSs in its cell. Nonetheless, the BSs are still in coordination to jointly manage the ICI. This approach, referred as 'interference coordination', has been recently investigated in [4–8]. These works tackled the multicell downlink beamforming problems to either jointly minimise the transmit power at the BSs or maximise the signal-to-interference-plus-noise ratios (SINR) at the MSs. In particular, Dahrouj and Yu [4] studied the optimal linear beamforming to either minimise the weighted sum transmit power or minimise the maximum per-antenna transmit power with guaranteed quality of service in terms of SINR at the MSs. In [5], a decentralised solution via dual decomposition was proposed to find the coordinated beamforming in minimising the sum transmit power at the BSs. Under the SINR maximisation criterion, the works in [6] made use of the bisection method and a second-order conic (SOC) solver to find the optimal solution. In [7], a direct solution approach was investigated to jointly maximise the SINR by the means of geometric programming (GP). More specifically, Cai *et al.* [7] studied the problem with one power constraint across the BSs, then generalised to the problem of per-BS power constraints. More recently, the connection between the power minimisation problem and the SINR maximisation problem was exploited to solve the SINR maximisation problem [8]. In [9], the coordinated multicell beamforming was studied with multiple BS assignment using long-term channel state information. It should be emphasised that these works only considered the multicell system with fixed BS-MS assignments.

In this work, we focus on the coordinated beamforming designs with dynamic BS-MS assignments. Since there is always a tradeoff between the level of coordination aganist the implementation complexity, it is desirable that a MS is only assigned to a subset of BSs. Thus, this raises the question of choosing the 'best' BSs for the coordinated transmissions to that particular MS, that is, which BSs should the MS be assigned to? This problem was recently investigated in [10], where a decentralised BS assignment algorithm with zero-forcing (ZF) precoding was proposed to minimise the sum transmit power at the BSs. However, the approach in [10] generally requires the number of MSs not exceeding the number of transmit antennas at 'each' BS such that the interference can be completely eliminated by ZF precoding. The algorithm proposed in [10] was also limited to the interference coordination mode only. A recent work in [11] examined the joint BS assignment and precoder design problem with the focus on the network sum-rate maximisation. Different from studies in [10, 11], this work considers the joint BS-MS assignment and beamforming design with the two following design objectives: (i) minimising the transmit power margin at the BSs and (ii) maximising the SINR margin at the MSs. It shall be shown that the joint beamforming design and BS assignment problems are integer programs, which are combinatorially hard. By the means of convex relaxation, these integer programs can be deduced into known assignment problems, where the efficient algorithms can be readily applied. This allows a simple method to solve these dynamic BS assignment problems with low complexity. Simulations show that the proposed relaxation-based assignment schemes, while being suboptimal, can outperform heuristic fixed assignment schemes, such as channel-based and location-based assignments.

Notations: Superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{*}$, $(\cdot)^{\mathrm{H}}$ stand for transpose, complex conjugate and complex conjugate transpose operations, respectively; upper-case bold face letters are used to denote matrices whereas lower-case bold face letters are used to denote column vectors; $\mathrm{diag}(d_1, d_2, \ldots, d_M)$ denotes an $M \times M$ diagonal matrix with diagonal elements $d_1, d_2, \ldots, d_M$; $[\cdot]_{i,j}$ denotes the (i, j) element of the matrix argument; x^* indicates the optimal value of the variable x.

2 System model

Consider a multiuser downlink beamforming system with Q coordinated cells operating on the same frequency channel while concurrently serving K MSs, as illustrated in Fig. 1. At each cell, a multiple-antenna BS multiplexes several user data streams in space, then simultaneously transmits them to its connected remote MSs. Herein, it is assumed that each BS is equipped with M antennas, whereas the MS is equipped with one antenna. In this work, we investigate the coordinated multicell system, where the BSs cooperate with each other to control both the signal transmission and the interference at each MS. In addition, the intended signal for a MS can be transmitted from one or more BSs, depending on the system design setting.

In the downlink transmission to a particular MS, say MS-i, its received signal y_i is given by

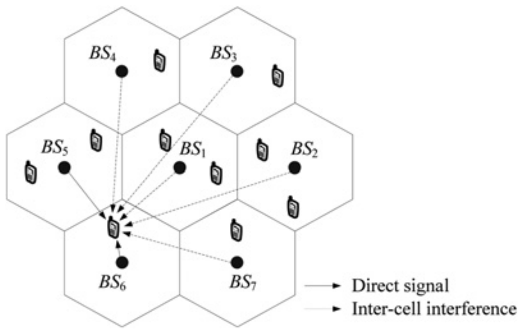

Fig. 1 *Seven-cell network with ten randomly located mobile stations*

$$y_i = \sum_{q=1}^{Q} \boldsymbol{h}_{q_i}^{\mathrm{H}} \boldsymbol{x}_{q_i} + \sum_{j \neq i}^{K} \sum_{q=1}^{Q} \boldsymbol{h}_{q_i}^{\mathrm{H}} \boldsymbol{x}_{q_j} + z_i \qquad (1)$$

where $\boldsymbol{x}_{q_i}$ is an $M \times 1$ complex vector representing the transmitted signal at BS-q for MS-i, $\boldsymbol{h}_{qi}^{*}$ is an $M \times 1$ complex channel vector from BS-q to MS-i, and z_i is the additive white Gaussian noise (AWGN) with the power spectral density σ^2. Let s_{q_i} be the variable indicating the assignment of MS-i to BS-q, where $s_{q_i} = 1$ if BS-q transmits data to MS-i, otherwise $s_{q_i} = 0$. In the coordinated beamforming design under consideration, the transmitted signal $\boldsymbol{x}_{q_i}$ can be represented in the form as $\boldsymbol{x}_{q_i} = s_{q_i} \boldsymbol{w}_{q_i} u_i$, where u_i is a complex scalar representing the signal intended for MS-i, and $\boldsymbol{w}_{q_i}$ is an $M \times 1$ beamforming vector for MS-i. Without loss of generality, let $\mathbb{E}[|u_i|] = 1$. It is easy to verify that the SINR at MS-i is

$$\mathrm{SINR}_i = \frac{\left| \sum_{q=1}^{Q} s_{q_i} \boldsymbol{h}_{q_i}^{\mathrm{H}} \boldsymbol{w}_{q_i} \right|^2}{\sum_{j \neq i}^{K} \left| \sum_{q=1}^{Q} s_{q_j} \boldsymbol{h}_{q_i}^{\mathrm{H}} \boldsymbol{w}_{q_j} \right|^2 + \sigma^2} \qquad (2)$$

At the transmitting end, the total transmit power at BS-q is then given by

$$P_q = \sum_{i=1}^{K} \mathbb{E}\left[\|\boldsymbol{x}_{q_i}\|^2 \right] = \sum_{i=1}^{K} s_{q_i}^2 \|\boldsymbol{w}_{q_i}\|^2 \qquad (3)$$

To this end, we shall investigate the coordinated beamforming designs in either jointly minimising the transmit power margin at the BSs subject to SINR constraints at the MSs or jointly maximising the achievable SINR at the MSs with individual power constraints at the BSs. We first make a brief revisit to these problems with pre-determined BS assignments, which will serve as an immediate step to the analysis of the dynamic BS assignment problems later on.

3 Coordinated downlink beamforming design with known BS assignments

In this section, let us first assume that each MS is already assigned to a particular set of serving BSs, that is, s_{q_i}'s are known. Given the maximum number of BSs to serve a MS, say MS-i, the assignment can be performed based on heuristic selection criteria, such as

- Location-based assignments: MS-i is assigned to the BS(s) which are the closest to it in physical distance.
- Channel-based assignments: MS-i is assigned to the BS(s) from which the downlink channel strengths are the strongest.

These assignment schemes are certainly the most straightforward options for connecting the MSs, especially the ones in the cell-edge region, to the best BS(s). With known s_{q_i}, the next task is to design beamforming vector in order to optimise a certain objective of the system. Specifically, we consider two design criteria: (i) jointly minimise the transmit power margin at the BSs

$$\mathcal{P}_1: \quad \underset{\boldsymbol{w}_{q_i}}{\text{minimise}} \ \max_q \frac{P_q}{P_q^{\max}} \quad (4)$$
$$\text{subject to } \text{SINR}_i \geq \gamma_i, \quad \forall i$$

where γ_i is the target SINR at MS-i, and $P_q^{\max}$ is the maximum power available at BS-q, and (ii) jointly maximise the achievable SINR margin at the MSs

$$\mathcal{S}_1: \quad \underset{\boldsymbol{w}_{q_i}}{\text{maximise}} \ \min_i \frac{\text{SINR}_i}{\gamma_i} \quad (5)$$
$$\text{subject to } P_q \leq P_q^{\max}, \quad \forall q$$

where γ_i is now treated as the weight factor for MS-i. Under the optimisation $\mathcal{P}_1$, the system tries to balance the power consumption at each BS and does not overuse any of them, whereas meeting the SINR requirements at all the MSs. On the contrary, under the optimisation S_1, the system tries to balance the achievable SINR at the MSs with strict power constraints at the BSs.

Denote $\boldsymbol{w}_i = \left[\boldsymbol{w}_{1_i}^{\mathrm{T}}, \ldots, \boldsymbol{w}_{Q_i}^{\mathrm{T}}\right]^{\mathrm{T}}$ as the beamformer from Q BSs to MS-i. Let $\boldsymbol{h}_{i,j} = \left[\text{diag}\left(s_{1_j}, \ldots, s_{Q_j}\right) \otimes \boldsymbol{I}_M\right] \times \left[\boldsymbol{h}_{1_i}^{\mathrm{T}}, \ldots, \boldsymbol{h}_{Q_i}^{\mathrm{T}}\right]^{\mathrm{T}}$, then $\left|\boldsymbol{h}_{i,j}^{\mathrm{H}}\boldsymbol{w}_j\right|^2$ is the effective interference power caused by MS-j's signal at MS-i. Let α be the margin of the transmit power at each BS to its maximum level, that is, $\alpha = \max_q P_q/P_q^{\max}$. The optimisation $\mathcal{P}_1$ is then equivalent to

$$\underset{\alpha, \{\boldsymbol{w}_i\}_{\forall i}}{\text{minimise}} \ \alpha \quad (6)$$
$$\text{subject to } \frac{\left|\boldsymbol{h}_{i,i}^{\mathrm{H}}\boldsymbol{w}_i\right|^2}{\sum_{j\neq i}^{K}\left|\boldsymbol{h}_{i,j}^{\mathrm{H}}\boldsymbol{w}_j\right|^2 + \sigma^2} \geq \gamma_i, \quad \forall i$$
$$\sum_{m=M(q-1)+1}^{Mq}\left[\sum_{i=1}^{K}\boldsymbol{w}_i\boldsymbol{w}_i^{\mathrm{H}}\right]_{m,m} \leq \alpha P_q^{\max}, \quad \forall q$$

where the summation $\sum_{m=M(q-1)+1}^{Mq}\left[\sum_{i=1}^{K}\boldsymbol{w}_i\boldsymbol{w}_i^{\mathrm{H}}\right]_{m,m}$ is the transmit power at the M antennas corresponding to BS-q.

It is noted that the BS assignment parameters s_{q_i}'s were removed from the power constraints in the restated problem (6). The reason is that if $s_{q_i} = 0$, $\boldsymbol{w}_{q_i} \neq \boldsymbol{0}$ does not affect the achievable SINR at MS-i while increasing the transmit power at BS-q, which ultimately increases the objective function. Thus, the optimisation will automatically set $\boldsymbol{w}_{q_i} = \boldsymbol{0}$ if $s_{q_i} = 0$. It is noted that problem (6) presents a generic formulation of the multicell beamforming optimisation for both the interference coordination and MIMO cooperation multicell systems. Under this formulation, problem (6) resembles the single-cell downlink problem with per-antenna power constraints in [12]. The differences here are the power constraints being applied to groups of antennas (corresponding to each BS) and the nominal channel vectors $\boldsymbol{h}_{j,i}$'s, which carry the BS assignment information of each MS. Although problem (6) is not a convex problem because of the inherently non-convex SINR constraints, it can be transformed into a convex SOC program. Thus, the proposed algorithm based on the fixed-point iteration in [12, 13] can be readily applied to efficiently solve problem (6) without a need of an external conic solver.

Similarly, by introducing an auxiliary variable τ representing the SINR margin, that is, $\tau = \text{SINR}_i/\gamma_i$, the joint SINR maximisation problem (5) can be reformulated as

$$\underset{\tau, \{\boldsymbol{w}_i\}_{\forall i}}{\text{maximise}} \ \tau$$
$$\text{subject to } \frac{\left|\boldsymbol{h}_{i,i}^{\mathrm{H}}\boldsymbol{w}_i\right|^2}{\sum_{j\neq i}^{K}\left|\boldsymbol{h}_{i,j}^{\mathrm{H}}\boldsymbol{w}_j\right|^2 + \sigma^2} \geq \tau\gamma_i, \quad \forall i$$
$$\sum_{m=M(q-1)+1}^{Mq}\left[\sum_{i=1}^{K}\boldsymbol{w}_i\boldsymbol{w}_i^{\mathrm{H}}\right]_{m,m} \leq P_q^{\max}, \quad \forall q \quad (7)$$

Like the power minimisation problem (6), the BS assignment parameters s_{q_i}'s are removed from the power constraints in the above problem with no effect on the optimal solution. However, unlike problem (6), problem (7) is non-convex, since there is no known method to transform the SINR constraints into convex ones with τ as a variable. Nonetheless, various approaches in literature were proposed to optimally solve this non-convex problem. The most straightforward approach is the bisection method which approximates the optimal τ^* by consecutively solving convex feasibility problems with varying SINR target τ [6]. However, this approach is rather unappealing since it requires many iterations in the bisection method as well as a standard conic solver. Alternately, this SINR minimax problem can be solved directly by utilising the fixed-point iteration in coupling with the projected gradient method [7, 8]. In fact, this approach can bypass the bisection method, and more importantly, the need of an external conic solver.

4 Joint BS assignment and coordinated downlink beamforming design

4.1 Problem formulations

In this section, we now proceed to examine the optimisation with joint beamforming and BS assignments. In particular, the BS assignments are now treated as variables that the system needs to optimise as well. Under the design criterion (i), the optimisation problem can be stated as

$$\mathcal{P}_2: \quad \underset{\boldsymbol{w}_{q_i}, s_{q_i}}{\text{minimise}} \ \max_q \frac{P_q}{P_q^{\max}} \quad (8a)$$

$$\text{subject to } \text{SINR}_i \geq \gamma_i, \quad \forall i \quad (8b)$$

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$$\sum_{q=1}^{Q} s_{q_i} = s_i^{\max}, \quad \forall i \tag{8c}$$

$$s_{q_i} = \{0, 1\}, \ \forall q, \ \forall i \tag{8d}$$

Similarly, under the design criterion (ii), the optimisation problem is

$$\mathcal{S}_2: \quad \underset{w_{q_i}, s_{q_i}}{\text{maximise}} \ \min_i \frac{\text{SINR}_i}{\gamma_i} \tag{9a}$$

$$\text{subject to } P_q \le P_q^{\max}, \quad \forall q \tag{9b}$$

$$\sum_{q=1}^{Q} s_{q_i} = s_i^{\max}, \quad \forall i \tag{9c}$$

$$s_{q_i} \in \{0, 1\}, \ \forall q, \ \forall i \tag{9d}$$

Unlike the optimisation problem $\mathcal{P}_1$ and S_1, the two additional constraints (8c) and (8d) in $\mathcal{P}_2$ and (9c) and (9d) in S_2 are included to dictate the maximum number of BSs, $s_i^{\max}$, to serve MS-i, and the assignment s_{q_i}'s as binary variables.

Remark 1: In the problem formulations (8) and (9), $s_i^{\max}$, representing the level of coordination among the BSs, can be set to any number between 1 and Q. When $s_i^{\max} = Q, \quad \forall i$, this effectively means that each MS can be served to all the BSs, that is, full MIMO cooperation mode. The two problems become the beamforming design problems $\mathcal{P}_1$ and S_1, respectively, with $s_{q_i} = 1, \ \forall q, \quad \forall i$. On the contrary, if $s_i^{\max} = 1, \quad \forall i$, the problems are equivalent to selecting one best BS for each MS in the interference coordination mode.

Remark 2: Owing to the binary constraints (8d) and (9d), problems $\mathcal{P}_2$ and S_2 are non-convex mixed integer programs, which are NP-hard [14]. Thus, the two problems are combinatorially hard with the worst case exponential complexity. An exhaustive search may be utilised to find their optimal solutions. However, the exhaustive search requires solving multiple optimisations $\mathcal{P}_1$ and S_1 corresponding to all of the $\binom{Q}{s_1^{\max}} \times \binom{Q}{s_2^{\max}} \times \cdots \times \binom{Q}{s_K^{\max}}$ possible combinations of s_{q_i}'s. Thus, this approach is clearly not viable for practical implementation. Other algorithms, such as the branch and bound algorithm, might be able to find the optimal solution to an integer program without the exhaustive search [14]. However, it is noted that such approaches are limited to a certain subclass of integer programs and might be very slow as well. In addition, it remains unknown whether those approaches can be applied to the mixed integer programs under consideration with the beamforming vector variables. Consequently, an efficient joint BS assignment and beamforming design algorithm with low complexity and near optimality is highly desirable. Using convex relaxation techniques, such an algorithm shall be investigated as next.

4.2 Convex relaxation to joint BS assignment and beamforming design

In this section, we consider a simple, yet efficient relaxation approach to solve the non-convex integer programs $\mathcal{P}_2$ and S_2. For an illustrative purpose, we only focus on solving the problem S_2e approach then can be easily adapted to solve problem $\mathcal{P}_2$. At first, we reformulate the optimisation problem such that some of the constraints can be devised into a convex form as follows

$$\underset{\tau, w_{q_i}, s_{q_i}}{\text{maximise}} \ \tau \tag{10a}$$

$$\text{subject to } \overline{\text{SINR}}_i \ge \tau\gamma_i, \quad \forall i \tag{10b}$$

$$\sum_{i=1}^{K} \left\| w_{q_i} \right\|^2 \le P_q^{\max}, \quad \forall q \tag{10c}$$

$$\left\| w_{q_i} \right\|^2 \le s_{q_i} P_q^{\max}, \quad \forall q, \forall i \tag{10d}$$

$$\text{and constraints (9c), (9d)} \tag{10e}$$

where τ again represents the minimum achievable SINR margin at the MSs. Different from SINR_i given in (2), $\overline{\text{SINR}}_i$ in constraint (10b) is now defined as

$$\overline{\text{SINR}}_i = \frac{\left| \sum_{q=1}^{Q} \boldsymbol{h}_{q_i}^{\text{H}} \boldsymbol{w}_{q_i} \right|^2}{\sum_{j \ne i}^{K} \left| \sum_{q=1}^{Q} \boldsymbol{h}_{q_i}^{\text{H}} \boldsymbol{w}_{q_j} \right|^2 + \sigma^2} \tag{11}$$

where the assignment variables s_{q_i}'s do not appear in the SINR equation. It is noted that constraint (10) relates the beamformer $\boldsymbol{w}_{q_i}$ to the BS assignment variable s_{q_i} in such a way that s_{q_i}'s are no longer needed in the new SINR formulation. To clarify this reformulation, if BS-q does not transmit information signal to MS-i, that is, $s_{q_i} = 0$, the constraint automatically enforces $\boldsymbol{w}_{q_i} = \boldsymbol{0}$. On the other hand, if $s_{q_i} = 1$, then the transmit power given for MS-i at BS-q, $\left\| \boldsymbol{w}_{q_i} \right\|^2$ must be less than $P_q^{\max}$. Thus, it can be concluded the optimisation problems (4.1) and (4.2) are indeed equivalent.

Second, to avoid the intractability of the integer program in problem (10), the non-convex constraints $s_{q_i} \in \{0, 1\}$ is replaced with the convex constraints $s_{q_i} \in [0, 1]$. We obtain the relaxation of problem (10) as follows

$$\underset{\tau, s_{q_i}, w_{q_i}}{\text{maximise}} \ \tau \tag{12a}$$

$$\text{subject to } \overline{\text{SINR}}_i \ge \tau\gamma_i, \quad \forall i \tag{12b}$$

$$\sum_{i=1}^{K} \left\| w_{q_i} \right\|^2 \le P_q^{\max}, \quad \forall q \tag{12c}$$

$$\left\| w_{q_i} \right\|^2 \le s_{q_i} P_q^{\max}, \quad \forall q, \forall i \tag{12d}$$

$$\sum_{q=1}^{Q} s_{q_i} \le s_i^{\max}, \quad 0 \le s_{q_i} \le 1, \quad \forall q, \forall i \tag{12e}$$

It is worth noting that the approach of relaxing an integer program into a convex program has been commonly considered in the literature, for example, see [15, 16, 17] and references therein. Although the relaxed problem (12) is yet to be convex, solving (12) is now a much easier task than solving the original problem (10). In particular, for a fixed τ, consider the following convex feasibility problem

$$\begin{aligned} &\text{find } \left\{\boldsymbol{w}_{q_i}\right\}_{\forall i, \forall q}, \ \left\{s_{q_i}\right\}_{\forall i, \forall q} \\ &\text{subject to } \sqrt{1+\frac{1}{\tau\gamma_i}}\sum_{q=1}^{Q}\boldsymbol{h}_{q_i}^{\mathrm{H}}\boldsymbol{w}_{q_i} \geq \left\| \begin{matrix} \sum_{q=1}^{Q}\boldsymbol{h}_{q_i}^{\mathrm{H}}\boldsymbol{w}_{q_1} \\ \vdots \\ \sum_{q=1}^{Q}\boldsymbol{h}_{q_i}^{\mathrm{H}}\boldsymbol{w}_{q_K} \\ \sigma \end{matrix} \right\|, \quad \forall i \qquad (13) \\ &\text{and constraints (12c)} - \text{(12e)} \end{aligned}$$

where the SINR constraints are recast into convex SOC forms. Consequently, the optimal solution to (12) can be readily obtained by the bisection method in conjunction with solving the convex feasibility problem (13).

It is noted that the optimal solution of the relaxation problem (12) is not necessarily equivalent to that of the original problem (10). In particular, while the optimal BS-MS assignments obtained from the relaxation problem (12) can be fractional, the BS-MS assignments in the original problem must be binary. However, one can take advantage of the relation between the two problems to approximate a suboptimal binary solution of problem (10). Denote $s_{q_i}^{\star}$, $s_{q_i}^{+}$ and $s_{q_i}^{-}$ as the optimal solutions of the original problem (10), the relaxation problem (12) and the approximated suboptimal solution, respectively. A straightforward way to obtain $s_{q_i}^{-}$ from $s_{q_i}^{+}$ is the 'rounding technique', which has been commonly employed to obtain sub-optimal solution from the relaxed optimal solution [15]. Specifically, from the solution set $\left\{s_{q_i}^{+}\right\}_{q=1}^{Q}$ corresponding to MS-i, choose the $s_i^{\max}$ largest values and round them to 1. That is, for MS-i, setting the $s_i^{\max}$ corresponding terms $s_{q_i}^{-}$ to 1, whereas the remaining elements are set to 0. After determining the suboptimal BS-MS assignments $s_{q_i}^{-}$ for each MS, one may proceed to the optimisation S_1 to determine to the beamformers. Hereafter, we refer this relaxation and rounding technique as the 'relaxation-based-1' scheme.

Denote τ^{+}, τ^{*} and τ^{-} as the optimal objective values obtained from the relaxed problem (12), the original problem (10), and the relaxation-based-1 scheme, respectively. Clearly, τ^{*} is upper-bounded by τ^{+} since the feasible set of the latter contains that of the former. On the other hand, τ^{*} is lower-bounded by τ^{-} as τ^{-} is obtained from a suboptimal solution. In general, one has

$$\tau^{-} \leq \tau^{*} \leq \tau^{+} \qquad (14)$$

The difference between τ^{+} and τ^{-} is called the 'relaxation' gap which is always non-negative. If it happens to be zero, the optimal value of the relaxation problem will be also optimal for the original program. However, because of relaxation to the integer constraints and the approximation in the 'rounding technique', the relaxed-based scheme, whereas being simple, may not result in a highly accurate solution. We break down the drawbacks of the proposed 'relaxation-based-1' schemes into the following remarks.

Remark 3: In a typical cellular system, the number of MSs is significantly larger than the number of BSs $(K \gg Q)$. At the optimal solution of problem (12), it is expected from constraint (12c) that $\|\boldsymbol{w}_{q_i}\|^2$ is in the order of $P_q^{\max}/K$. On the other hand, constraint (12e) enforces s_{q_i} in the order of $s_i^{\max}/Q$. Thus, constraint (12d) is generally loose. Consequently, both constraints (12) and (12e) have little impact on the optimal solution of problem (12). As a result, the largest terms in $s_{q_i}^{+}$ might not always be a good indicator for assigning the 'best' BS(s) to MS-i.

Remark 4: In problem (12), because of constraint (12d), the assignment variable s_{q_i} literally indicates the transmit power for MS-i at BS-q, that is, $\left\|\boldsymbol{w}_{q_i}\right\|^2$. Thus, the rounding step of the $s_i^{\max}$ largest elements of $s_{q_i}^{+}$ to 1 to obtain $s_{q_i}^{-}$ can be interpreted as assigning the $s_i^{\max}$ out of the Q BSs that are transmitting the highest power levels to MS-i. However, as indicated in the SINR formulation in (11), the transmit power $\left\|\boldsymbol{w}_{q_i}\right\|^2$ does not actually contribute the achievable SINR at MS-i as the receive power $\left|\boldsymbol{w}_{q_i}^{\mathrm{H}}\boldsymbol{h}_{q_i}\right|^2$. Intuitively, which BS is assigned to serve MS-i should be based on the merit of its beamformer $\boldsymbol{w}_{q_i}$ being aligned to the channel to MS-i in order to maximise the receive power $\left|\boldsymbol{w}_{q_i}^{\mathrm{H}}\boldsymbol{h}_{q_i}\right|^2$.

Remark 5: In the proposed 'relaxation-based-1' scheme, the relaxation problem (12) needs to be solved first to obtain $s_{q_i}^{+}$. In that process, we may need to use an external optimisation software package to solve the convex feasibility (13), in conjunction with the bisection method. However, this approach is highly time-consuming since it requires many iterations in the bisection method as well as the external optimisation solver. Note that the efficient algorithm in solving problem S_1 mentioned in Section 3 is not applicable to problem (12) because of the presence of the assignment variables s_{q_i}'s.

To address the drawbacks in implementing the 'relaxation-based-1' scheme, we propose an alternative relaxation scheme for solving problem (10), namely 'relaxation-based-2' scheme, as follows:

1. *Relaxation step:* solve the relaxed problem (12) without constraints (12d) and (12e). This is equivalent to solving problem $\mathcal{S}_1$ with $s_{q_i} = 1, \quad \forall i, \forall q$, that is, optimising the beamformers $\boldsymbol{w}_{q_i}$ in

$$\begin{aligned} &\underset{\boldsymbol{w}_{q_i}}{\text{maximise}} \ \min_i \frac{\overline{\mathrm{SINR}}_i}{\gamma_i} \\ &\text{subject to } P_q \leq P_q^{\max}, \quad \forall q \end{aligned} \qquad (15)$$

In this case, the efficient iterative algorithm mentioned in Section 3 to solve problem S_1 is readily applicable. Let us denote the obtained solution as $\tilde{\boldsymbol{w}}_{q_i}, \forall i, \forall q$.

2. *Rounding step:* for each MS, say MS-i, calculate the Q terms $\left|\tilde{\boldsymbol{w}}_{q_i}^{\mathrm{H}}\boldsymbol{h}_{q_i}\right|$. Out of the obtained Q terms, find the $s_i^{\max}$ largest terms and set the corresponding BS-MS assignment variables $s_{q_i}^{-}$ to 1. The remaining elements in $s_{q_i}^{-}$ are set to 0.

After this rounding and assigning step, one may proceed to solve the optimisation problem S_1 with known BS

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assignments $s_{q_i}^-$ to obtain the optimal beamformer $\boldsymbol{w}_{q_i}$ for the 'relaxation-based-2' scheme.

It is noted that relaxation and rounding approaches can be employed to find suboptimal solutions to the power minimisation problem $\mathcal{P}_2$ in a similar manner. We should stress here that more efficient algorithms in terms of performance and/or complexity are also possible. However, an investigation for 'better' algorithms is an interesting research direction but beyond the scope of this paper.

4.3 Complexity comparison of the proposed schemes

This section is to address the complexity in implementing the proposed relaxation-based BS-MS assignment schemes, in comparison to the heuristic ones (channel-based and location-based). The problem of SINR margin maximisation is considered as an example. With the heuristic BS-MS assignment schemes, the optimal beamformers to problem S_1 can be found by the SOC optimisation technique with polynomial complexity [7, 8]. With the 'relaxation-based-2' scheme, because of the two-step optimisation procedure, one needs to solve the problem S_1 twice, one with the assignments s_{q_i}'s all set to 1 and one after the rounding step. The 'relaxation-based-2' scheme also requires some simple operations at the rounding step to designate the BS-MS assignments. Thus, its complexity is about twice that of the 'channel-based' or 'location-based' scheme. On the other hand, the 'relaxation-based-1' scheme requires an external optimisation software package to solve multiple convex feasibility problems (12) because of the bisection method and one instance of solving problem S_1 after the rounding step. As a result, the implementation of the first relaxation-based scheme is more computationally complex and time-consuming than second scheme.

5 Numerical simulations

This section presents some numerical evaluations on the power consumption and the achievable SINR in a multiuser multicell system employing coordinated beamforming with per-BS power constraints. We compare the results obtained from the relaxation-based BS assignment schemes to that obtained from the heuristic BS assignment ones, that is, channel-based and location-based. Since the algorithms presented in Section 3 can only be applied to the 'relaxation-based-2' scheme, the results for the first relaxation-based scheme are obtained from the external convex optimisation package cvx [18]. We consider a 7-cell system with 10 MSs, as illustrated in Fig. 1, unless stated otherwise. Each BS is equipped with four antennas and each MS is served by a maximum two BSs. Assuming that the locations of the BSs are fixed and the distance between two nearest BS is normalised to one. On the contrary, the location of each MS is randomly generated across the multicell network. The channel coefficients, depending on the distance between each BS-MS pair, are then generated from independent and identically distributed (i.i.d). Gaussian random variables using the path loss model with the path loss exponent of 3 and the reference distance of 1. The same power constraint $P_q^{\max}$ is imposed at each BS, whereas the same target SINR γ_i is set at each MS. The AWGN power spectral density σ^2 is assumed to be 0.01.

Fig. 2 illustrates the transmit power margin α to the power limit $P_q^{\max}$ with different target SINRs τ_i's. Herein, the power

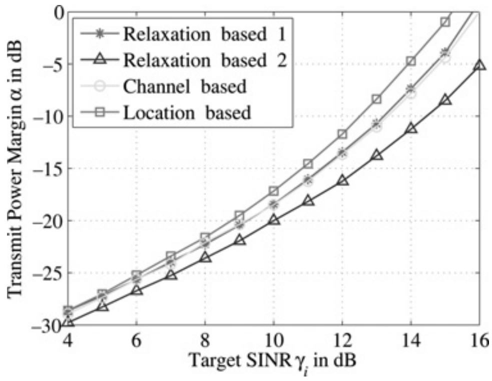

Fig. 2 *Average transmit power margin α to the power limit at each BS against the target SINR γ_i's at MSs*

limit $P_q^{\max}$ is set at 1, such that the power limit is set to be 20 dB higher than the AWGN power level. As the target SINR varies, 10 000 channel realisations at each SINR value are used to obtain the average transmit power margin α in Fig. 2. As shown in the figure, as the target SINR increases, the required transmit power to meet the target SINR also becomes larger. Out of the considered BS assignment schemes, it is observed that the 'relaxation-based-2' scheme significantly outperforms the other schemes. In particular, this dynamic scheme can save the transmit power at each BS up to 5 and 4 dB over then location-based and channel-based schemes, respectively. On the other hand, the performance of the first relaxation and rounding scheme is at least comparable to the two heuristic ones.

Fig. 3 displays the average achievable SINR margin τ to the target SINR of 10 dB at the MSs against the transmit power limit at each BS in term of $P_q^{\max}/\sigma^2$. Clearly, increasing the transmit power at BS results in higher achievable SINR at the MSs. As expected from the figure, using the 'relaxation-based-2' scheme can further boosts the achievable SINR margin by nearly 1 dB, compared to the other three assignment schemes.

Table 1 presents an example of BS assignments for a 'specific channel realisation' with different assignment schemes. In this example, the target SINR γ_i is set at 10 dB, whereas the transmit power is limited such that $P_q^{\max}/\sigma^2 = 10$ dB. It is observed that depending on the design criterion, the relaxation and rounding schemes select different BS combinations for each MS to obtain an approximate optimal selection scheme. It should be

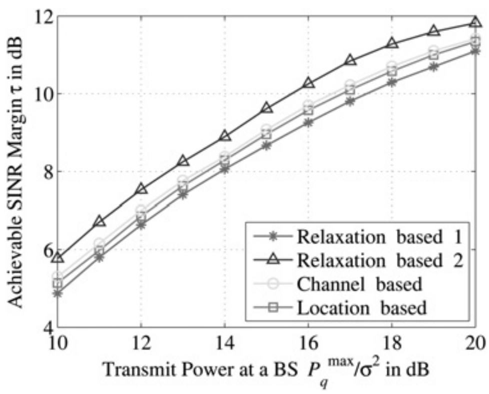

Fig. 3 *Achievable SINR margin τ at the MS against the power constraints at each BS*

Table 1 BS assignments with various schemes

Assignment schemes		BS assignments										Power margin in dB	SINR margin in dB
		MS 1	MS 2	MS 3	MS 4	MS 5	MS 6	MS 7	MS 8	MS 9	MS 10		
relaxation-based-1	power min.	(1,4)	(4,5)	(5,6)	(2,3)	(1,3)	(2,7)	(2,3)	(1,3)	(6,7)	(2,7)	– 11.5232	N/A
	SINR max.	(1,4)	(4,5)	(5,6)	(2,3)	(1,3)	(2,7)	(1,3)	(1,3)	(6,7)	(2,7)	N/A	5.8045
relaxation-ased-2	power min.	(1,4)	(4,5)	(1,6)	(1,3)	(1,3)	(6,7)	(1,3)	(1,4)	(6,7)	(2,7)	– 12.1996	N/A
	SINR max.	(1,4)	(4,5)	(1,6)	(1,3)	(1,3)	(6,7)	(1,3)	(1,4)	(6,7)	(2,7)	N/A	5.9627
channel-based		(1,4)	(1,7)	(1,6)	(1,3)	(1,3)	(6,7)	(1,3)	(1,7)	(6,7)	(2,7)	– 11.0634	5.3965
location-based		(1,4)	(1,4)	(1,6)	(2,3)	(1,3)	(1,7)	(1,3)	(1,3)	(6,7)	(2,7)	– 11.3895	5.1617

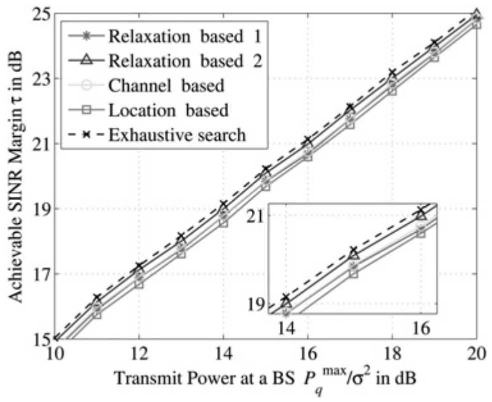

Fig. 4 *Achievable SINR margin τ at the MS against the power constraints at each BS with 3 BSs and 4 MSs*

emphasised that the dynamic assignment schemes perform much better than the static ones in both two design criteria. In this particular example, for the SINR maximisation criterion, the SINR margins are 5.8045 and 5.9624 dB for the relaxation-based-1 and -2 schemes, respectively, whereas the channel-based and location-based schemes only achieve the SINR margins of 5.3965 and 3.1617 dB, respectively.

Finally, Fig. 4 compares the performances between the proposed dynamic BS-MS assignment schemes with the exhaustive search. Since the simulation setup in Fig. 1 would take 21^{10} combinations for the BS-MS assignments, it is impossible to assess the optimal performance by the exhaustive search. In this simulation, the multicell network configuration is rather simple with 3 BSs and 4 randomly located MSs, such that only 3^4 different instances of BS-MS assignments are need for the exhaustive search. As indicated in the new simulation, the performance gap between the proposed 'relaxation-based-2' technique to the optimal one is rather small. The dynamic relaxation-based BS-MS assignment scheme performs about half-way between optimal exhaustive search technique and the heuristic schemes.

6 Conclusion

This paper studied efficient algorithms in coordinated multicell downlink beamforming with dynamic BS assignment consideration. We examined two the CoMP designs under the criteria: (i) minimising transmit power margin at the BSs with guaranteed SINR at each MSs and (ii) jointly maximising the minimum SINR margin at the MS. The two problems were initially studied with static and pre-determined BS assignments. We then examined the two problems with the BS assignments as variables to be optimised as well. It was shown that the joint beamforming and BS assignments problems are non-convex integer programs, which are combinatorially hard. Relying on convex relaxation techniques, this paper proposed efficient algorithms to solve the problems with low complexity. Simulations then showed the benefit of applying the dynamic BS assignment over the heuristic ones.

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