Optimal Uplink and Downlink Channel Assignment in a Full-Duplex Multiuser System

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***Abstract*—Full-duplex (FD) has emerged as a promising solution for increasing the data rate of wireless communication systems. With FD, a wireless terminal can transmit and receive concurrently at the same frequency band. This paper focuses on the resource allocation in a FD multiuser system. With a FD-enabled base-station (BS) and multiple half-duplex (HD) mobile stations (MS), we are interested in jointly optimizing the uplink and downlink channel assignment for each MS and maximizing the system sum-rate. Since the joint optimization problem is a difficult nonconvex problem, we then propose an iterative algorithm to obtain at least a locally optimal solution. In the proposed algorithm, the system sum-rate is maximized via an equivalent problem of minimizing the weighted sum mean-squared error, whereas the channel assignment is updated by a gradient method. Simulation results show that the FD mode has the potential to substantially enhance a multiuser system's data-rate, compared to the HD mode.**

I. Introduction

The ubiquitous deployment of wireless devices has resulted in ever-increasing demands on higher wireless system capacity. Expecting significant capacity increase in the next generation wireless system [1], various technologies have been proposed to improve the spectral efficiency given thee limited wireless spectrum. Among these technologies, full-duplex (FD) has recently attracted a lot of research attention in both academia and industry [2]. In half-duplex (HD), a wireless terminal can only transmit or receive at a particular frequency band. In contrast, in FD, two communicating wireless terminals can transmit/receive from the other on the same frequency band. Thus, FD is a promising solution which potentially doubles the spectral efficiency relatively to the HD mode [2], [3].

Until recently, FD has not been widely implemented due to the harmful effects of self-interference (SI). SI is caused by the transmitted signal of a terminal to itself, which then corrupts the desired signal being received by the terminal. In fact, due to proximity of the terminal's transmit and receive components, the SI is much higher than the thermal noise floor. Note that the doubled spectral efficiency promised by FD can only be obtained if the self-interference is fully suppressed to the noise floor. Recent advances in FD radios have combined various techniques, including antenna design, analog cancellation and digital cancellation, to effectively reduces the SI [2], [4]. It was shown experimentally in [5] that SI can be adequately suppressed to a certain level at which FD systems outperform HD systems in terms of data rate. In [6], a combination of analog and digital cancellation techniques were implemented and reported to effectively reduce the SI to the receiver noise floor. Such an achievement [6] was manageable with the FD terminal transmitting at 20 dBm, which is applicable for short-range systems, such as small-cell and WiFi. However, when a FD terminal transmits at 40 dBm, which is the case for cellular networks, full suppression of the SI is still a challenging task. Several factors, such as scatterings in the loopback channel and the limited dynamic range of the analog-to-digital converter (ADC), may limit the effectiveness of SI cancellation. The residual SI then becomes the limiting factor of FD radios.

In this work, we are interested in examining the resource allocation in a FD cellular system with multiple users. In the considered system, a FD-enabled base-station (BS) is simultaneously serving multiple HD mobile-stations (MS) in both uplink (UL) and downlink (DL) directions. Recent works in [7], [8] have studied a FD multiple-input multiple-output (MIMO) system under a similar setting where the MSs are divided into two groups: UL users and DL users. Based on convex approximation techniques, the numerical methods were then proposed to maximize the system spectral efficiency [7], [8] and energy efficiency [7]. Numerical results in [7] indicated that better spectral efficiency is achievable when the SI is sufficiently small. A more recent paper [9] investigated the problem of joint duplex mode selection, channel allocation, and power control to maximize the sum-rate of a FD femtocell network. A greedy channel allocation algorithm was proposed in [9] to schedule each channel a pair of users, one using the channel for UL transmission while the other using for the DL transmission. Different to the works in [7], [8], this paper considers the joint sum-rate maximization in both UL and DL directions, in addition to optimizing the channel assignment for each user. Also different to the work [9], the presented study considers the MIMO processing capability at the BS, which then allows multiple MSs to be scheduled in both UL and DL directions on a same frequency band simultaneously.

The organization of the paper is as follows. In Section II, we formulate the joint channel assignment and sum-rate maximization problem, which is shown to be a mixed-integer program. In Section III, by first fixing the channel assignment, the system sum-rate is maximized via an equivalent problem of minimizing the weighted sum mean-squared error (MMSE). Combined with the channel assignment update developed in Section IV, a locally optimal solution is then obtained for the original optimization problem. Simulation results presented in Section V show a substantial improvement in the system sum-rate in the FD mode, relatively to the HD mode.

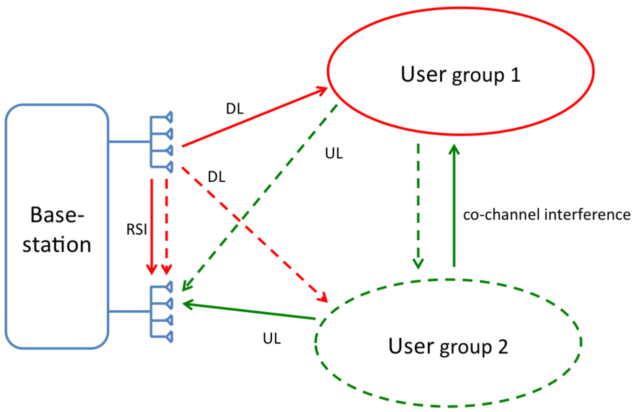

Fig. 1. Diagram of a multiuser system with a FD-enabled BS. Solid and dashed-dotted arrows denote the transmission on CH-1 and CH-2, respectively.

II. System Model and Problem Formulation

A. System Model

We consider a multiuser system with one $2M$-antenna BS concurrently serving K single-antenna MSs. It is assumed that the MSs can only function in the HD mode due to the limitations in the MSs' circuitry and size. Thus, each MS has to access two orthogonal channels (CH), one for its UL transmission and another for its DL reception. In this work, we assume that there are two CHs, denoted as CH-1 and CH-2, available for the DL/UL transmissions.

We denote $\mathcal{S}_1$ as the set of MSs which uses the DL transmission on CH-1. Likewise, denote $\mathcal{S}_2$ as the set of MSs who uses the DL transmission on CH-2. Certainly, one has $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$ due to the HD capability of the MSs and $\mathcal{S}_1 \cup \mathcal{S}_2 = \mathcal{K}$, where $\mathcal{K}$ is the set of MSs. As illustrated in Fig. 1, the MSs are divided into two groups, $\mathcal{S}_1$ and $\mathcal{S}_2$, accordingly to its DL channel. Note that there is no specific pre-determination on UL/DL channel for the MSs.

It is assumed that the BS is capable of operating in FD mode, where M antennas are used for transmission and the other M antennas are used for reception. When all the MSs transmit on the same CH and receive on the other CH, e.g., $\mathcal{S}_1 = \mathcal{K}$ and $\mathcal{S}_2 = \emptyset$, the system is said to be in the "HD mode". On the other hand, if $\mathcal{S}_1 \neq \emptyset$ and $\mathcal{S}_2 \neq \emptyset$, the system is said to be operating in the "FD mode". In this case, the BS is receiving and transmitting on both CHs. As depicted in Fig. 1, the UL transmission from one group of MSs may be corrupted by the residual self-interference (RSI) at the BS due to the DL transmission to the other group. Similarly at the MSs, the UL transmission from one group of MSs may cause additional co-channel interference to the other group.

Let us denote the DL and UL channels to/from MS-i on CH-c as $\mathbf{h}_{i,c}^*$ and $\mathbf{g}_{i,c}$. The loopback channel at the BS on CH-c is denoted as $\hat{\mathbf{H}}_c \in \mathbb{C}^{K\times K}$, whereas the co-channel interference link from MS-i to MS-j on CH-c is denoted as $\hat{g}_{ij,c}$. We first consider the DL transmission where the BS forms its transmitted signal in CH-c as follows:

$$\mathbf{x}_c = \sum_{i=1}^{K} \mathbf{w}_{i,c} s_i^{\mathrm{DL}}, \quad c = 1, 2, \tag{1}$$

where s_i^{DL} is the data symbol intended for MS-i and $\mathbf{w}_{i,c} \in \mathbb{C}^K$ is the DL beamformers on CH-c. Note that $\mathbf{w}_{i,c}$ is set to $\mathbf{0}$ if $i \notin \mathcal{S}_c$.

Should MS-i receive its DL transmission from the BS on CH-c, its received signal can be modeled as

$$\begin{aligned} y_{i,c}^{\mathrm{DL}} = & \underbrace{\mathbf{h}_{i,c}^H \mathbf{w}_{i,c} s_i^{\mathrm{DL}}}_{\text{signal}} + \underbrace{\mathbf{h}_{i,c}^H \sum_{j\neq i}^{K} \mathbf{w}_{j,c} s_j^{\mathrm{DL}}}_{\text{DL co-channel interference}} \\ & + \underbrace{\sum_{j=1}^{K} \hat{g}_{ji,c} \sqrt{p_{j,c}^{\mathrm{UL}}} s_j^{\mathrm{UL}}}_{\text{UL co-channel interference}} + z_{i,c}^{\mathrm{DL}}, \end{aligned} \tag{2}$$

where $p_{j,c}^{\mathrm{UL}}$ is the transmit power of MS-j on CH-c and $z_{i,c}^{\mathrm{DL}}$ is the AWGN with a power spectral density of σ^2. The MS then applies a scalar equalizer $\delta_{i,c}$ to its received signal such that its estimated symbol $\hat{s}_{i,c}^{\mathrm{DL}} \triangleq \delta_{i,c} y_{i,c}^{\mathrm{DL}}$. The average receive signal-to-noise-plus interference (SINR) at MS-i on CH-c is given by

$$\gamma_{i,c}^{\mathrm{DL}} = \frac{\left|\mathbf{h}_{i,c}^H \mathbf{w}_{i,c}\right|^2}{\sum_{j\neq i}^{K} \left|\mathbf{h}_{i,c}^H \mathbf{w}_{j,c}\right|^2 + \sum_{j=1}^{K} |\hat{g}_{ji,c}|^2 p_{j,c}^{\mathrm{UL}} + \sigma^2}. \tag{3}$$

We now consider the UL transmission from the same MS-i. The received signal at the BS on CH-$\hat{c}$, where $\hat{c} \triangleq 3 - c$, can be modeled as

$$\mathbf{y}_{\hat{c}}^{\mathrm{UL}} = \underbrace{\sum_{i=1}^{K} \mathbf{g}_{i,\hat{c}} \sqrt{p_{i,\hat{c}}^{\mathrm{UL}}} s_i}_{\text{signal from } K \text{ MSs}} + \underbrace{\hat{\mathbf{H}}_{\hat{c}} \mathbf{x}_{\hat{c}}^{\mathrm{DL}}}_{\text{RSI}} + \mathbf{z}_{\hat{c}}^{\mathrm{UL}}, \tag{4}$$

where $\hat{\mathbf{H}}_{\hat{c}} \mathbf{x}_{\hat{c}}^{\mathrm{DL}}$ represents the residual self-interference (RSI) on CH-$\hat{c}$. We assume that this loopback channel $\hat{\mathbf{H}}_c$ is unknown at the BS and i.i.d. with $\mathcal{CN}(0, \hat{\sigma}^2)$. Note that $p_{i,\hat{c}}^{\mathrm{UL}}$ should be set to 0 if $i \notin \mathcal{S}_c$. To decode the signal from MS-i, the BS then applies a receive beamformer $\mathbf{v}_{i,c}^H$ to equalize the received signal for MS-i on CH-$\hat{c}$ such as

$$\begin{aligned} \hat{s}_i^{\mathrm{UL}} = & \mathbf{v}_{i,\hat{c}}^H \mathbf{g}_{i,\hat{c}} \sqrt{p_{i,\hat{c}}^{\mathrm{UL}}} s_i + \mathbf{v}_{i,\hat{c}}^H \sum_{j\neq i}^{K} \mathbf{g}_{j,\hat{c}} \sqrt{p_{j,\hat{c}}^{\mathrm{UL}}} s_j \\ & + \mathbf{v}_{i,\hat{c}}^H \hat{\mathbf{H}}_{\hat{c}} \sum_{j=1}^{K} \mathbf{w}_{j,\hat{c}} s_j^{\mathrm{DL}} + \mathbf{v}_{i,\hat{c}}^H \mathbf{z}_{\hat{c}}^{\mathrm{UL}}. \end{aligned} \tag{5}$$

The average RSI power at the BS in the reception of MS-i's signal on CH-$\hat{c}$ can be calculated as

$$\begin{aligned} \mathrm{RSI}_{i,\hat{c}}(\mathbf{v}_{i,\hat{c}}) &= \mathbb{E}_{\hat{\mathbf{H}}_{\hat{c}}, s_j^{\mathrm{DL}}} \left[\left| \mathbf{v}_{i,\hat{c}}^H \hat{\mathbf{H}}_{\hat{c}} \sum_{j=1}^{K} \mathbf{w}_{j,\hat{c}} s_j^{\mathrm{DL}} \right|^2 \right] \\ &= \sum_{j=1}^{K} \mathbb{E}_{\hat{\mathbf{H}}_{\hat{c}}} \left[\left| \mathbf{v}_{i,\hat{c}}^H \hat{\mathbf{H}}_{\hat{c}} \mathbf{w}_{j,\hat{c}} \right|^2 \right] \\ &= \hat{\sigma}^2 \|\mathbf{v}_{i,\hat{c}}\|^2 \sum_{j=1}^{K} \|\mathbf{w}_{j,\hat{c}}\|^2. \end{aligned} \tag{6}$$

Thus, the average receive SINR for the uplink transmission from MS-i on CH-$\hat{c}$ is given by

$$\gamma_{i,\hat{c}}^{\rm UL} = \frac{p_{i,\hat{c}}^{\rm UL}\left|\mathbf{v}_{i,\hat{c}}^H\mathbf{g}_{i,\hat{c}}\right|^2}{\sum_{j\neq i}^{K} p_{j,\hat{c}}^{\rm UL}\left|\mathbf{v}_{i,\hat{c}}^H\mathbf{g}_{j,\hat{c}}\right|^2 + {\rm RSI}_{i,\hat{c}}(\mathbf{v}_{i,\hat{c}}) + \sigma^2\|\mathbf{v}_{i,\hat{c}}\|^2}. \quad (7)$$

Let $a_{i,c}$ be a binary variable indicating the association of MS-i to the set $\mathcal{S}_c$. More specifically, if $a_{i,1} = 1$ then MS-i belongs to the set $\mathcal{S}_1$. The sum UL and DL data rates of MS-i is then given by

$$R_i^{\rm DL+UL} = \sum_{c=1}^{2}\left[a_{i,c}\log\left(1+\gamma_{i,c}^{\rm DL}\right)+a_{i,\hat{c}}\log\left(1+\gamma_{i,c}^{\rm UL}\right)\right]. \quad (8)$$

In order to jointly optimize the channel assignment, the DL and UL beamformers, and the UL power control, we consider the following optimization

$$\begin{aligned} \underset{a_{i,c},\mathbf{w}_{i,c},\mathbf{v}_{i,c},p_{i,c}^{\rm UL}}{\text{maximize}} \quad & \sum_{i=1}^{K} R_i^{\rm DL+UL} & (9)\\ \text{subject to} \quad & p_{i,1}^{\rm UL} + p_{i,2}^{\rm UL} \leq p_{i,\max}^{\rm UL},\ \forall i, \\ & \sum_{i=1}^{K}\left[\|\mathbf{w}_{i,1}\|^2 + \|\mathbf{w}_{i,2}\|^2\right] \leq p_{0,\max}^{\rm DL}, \\ & a_{i,c}\in\{0,1\};\ a_{i,1}+a_{i,2}=1;\ \forall i. \end{aligned}$$

In the above optimization problem, $p_{i,\max}^{\rm UL}$ and $p_{0,\max}^{\rm DL}$ are the transmit power budgets at MS-i and the BS, respectively. The last constraint is to impose the association of a MS to only one set $\mathcal{S}_1$ or $\mathcal{S}_2$ at any time. Note that if $a_{i,c} = 0$, the optimization will automatically set $\gamma_{i,c}^{\rm DL} = 0$ and $\gamma_{i,\hat{c}}^{\rm UL} = 0$, i.e., $\mathbf{w}_{i,c} = \mathbf{0}$ and $p_{i,\hat{c}}^{\rm UL} = 0$. It is further observed that the optimization problem (9) is a nonconvex mixed integer program, which is NP-hard [10] in general. Inspired by a recent work in joint BS selection and beamforming design [11], an iterative algorithm is developed in the subsequent sections to obtain locally optimal solution to problem (9).

III. Sum-rate Maximization with Known Channel Assignment

A. The Optimization Problem

In this section, we first investigate a numerical method to solve problem (9) when $a_{i,c}$'s are known. At one extreme where $a_{i,c} = 1, \forall i$ and $a_{i,\hat{c}} = 0, \forall i$, i.e., the HD mode, the problem (9) becomes decoupled into two separate problems: one for the DL and one for the UL. The two problems, even being nonconvex, can be efficiently solved by converting them to the equivalent MSE minimization problems [12], [13]. Hereafter, we are interested in solving problem (9) for general cases with $0 \leq a_{i,c} \leq 1$. In case $0 < a_{i,1} < 1$, MS-i is said to use time sharing to access the DL channel on CH-1 by a split of $a_{i,1}$ time slot and on CH-2 by a split of $a_{i,2} = 1 - a_{i,1}$ time slot. With known $a_{i,c}$'s, the optimization can be restated as

$$\begin{aligned} \underset{\mathbf{w}_{i,c},\mathbf{v}_{i,c},p_{i,c}^{\rm UL}}{\text{maximize}} \quad & \sum_{i=1}^{K} R_i^{\rm DL+UL} & (10)\\ \text{subject to} \quad & p_{i,1}^{\rm UL} + p_{i,2}^{\rm UL} \leq p_{i,\max}^{\rm UL},\quad \forall i, \\ & \sum_{i=1}^{K}\left[\|\mathbf{w}_{i,1}\|^2 + \|\mathbf{w}_{i,2}\|^2\right] \leq p_{0,\max}^{\rm DL}. \end{aligned}$$

It is noted that the above optimization is still not a nonconvex problem due to the appearance of the optimization variables in the denominators of $\gamma_{i,c}^{\rm DL}$ and $\gamma_{i,c}^{\rm UL}$. By applying the weighted MMSE framework proposed in [13], problem (10) can be transformed into the following weighted MMSE problem

$$\begin{aligned} \underset{\substack{\omega_{i,c}^{\rm DL},\delta_{i,c},\mathbf{w}_{i,c}\\ \omega_{i,c}^{\rm UL},\mathbf{v}_{i,c},p_{i,c}^{\rm UL}}}{\text{minimize}} \quad & \sum_{i=1}^{K}\sum_{c=1}^{2}\Big[a_{i,c}\left(\omega_{i,c}^{\rm DL}e_{i,c}^{\rm DL} - \log\omega_{i,c}^{\rm DL}\right) \\ & \qquad + a_{i,\hat{c}}\left(\omega_{i,c}^{\rm UL}e_{i,c}^{\rm UL} - \log\omega_{i,c}^{\rm UL}\right)\Big] & (11)\\ \text{subject to} \quad & p_{i,1}^{\rm UL} + p_{i,2}^{\rm UL} \leq p_{i,\max}^{\rm UL},\ \forall i, \\ & \sum_{i=1}^{K}\left[\|\mathbf{w}_{i,1}\|^2 + \|\mathbf{w}_{i,2}\|^2\right] \leq p_{0,\max}^{\rm DL}, \end{aligned}$$

where $e_{i,c}^{\rm UL} \triangleq \mathbb{E}\left\{\left|\hat{s}_i^{\rm UL} - s_i^{\rm UL}\right|^2\right\}$ and $e_{i,c}^{\rm DL} \triangleq \mathbb{E}\left\{\left|\hat{s}_i^{\rm DL} - s_i^{\rm DL}\right|^2\right\}$ are defined as the MSE of the data symbols transmitted from/to MS-i, respectively; $\omega_{i,c}^{\rm UL}$ and $\omega_{i,c}^{\rm DL}$ are the weights associated with the MSE values $e_{i,c}^{\rm UL}$ and $e_{i,c}^{\rm DL}$, respectively. These MSE weights are to be optimized in problem (11) as well.

Proposition 1. *The sum-rate maximization problem* (10) *is equivalent to the weighted sum-MSE minimization problem* (11)*, where a locally optimal solution to latter is also locally optimal to the former.*

Proof: The proof for this proposition is similar to that in [13] for the case of the DL multiuser system. We omit the details for brevity. ∎

B. The Weighted MMSE Algorithm

Having the equivalence between problems (10) and (11), we proceed to solve the latter. It is noted that problem (11) is not a jointly convex problem. However, it is convex in each set of variables $\omega_{i,c}^{\rm DL}$'s, $\delta_{i,c}$'s, $\mathbf{w}_{i,c}$'s, $\omega_{i,c}^{\rm UL}$'s, $\mathbf{v}_{i,c}$'s, and $p_{i,c}^{\rm UL}$'s. Thus, a local optimal solution to the problem can be found by alternately optimizing each variable set, while keeping the other sets fixed. To this end, we present the framework to obtain a close-formed solution for each variable set.

By expanding the MSE value $e_{i,c}^{\rm UL}$ and $e_{i,c}^{\rm DL}$, one has

$$\begin{aligned} e_{i,c}^{\rm UL} &= \left|1 - \sqrt{p_{i,c}^{\rm UL}}\mathbf{v}_{i,c}^H\mathbf{g}_{i,c}\right|^2 + \sum_{j\neq i}^{K} p_{j,c}^{\rm UL}\left|\mathbf{v}_{i,c}^H\mathbf{g}_{j,c}\right|^2 \\ &\quad + \left(\sum_{j=1}^{K}\|\mathbf{w}_{j,c}\|^2\hat{\sigma}^2 + \sigma^2\right)\|\mathbf{v}_{i,c}\|^2, \end{aligned} \quad (12)$$

$$\begin{aligned} e_{i,c}^{\rm DL} &= \left|1 - \delta_{i,c}\mathbf{h}_{i,c}^H\mathbf{w}_{i,c}\right|^2 + \sum_{j\neq i}^{K}\left|\delta_{i,c}\mathbf{h}_{i,c}^H\mathbf{w}_{j,c}\right|^2 \\ &\quad + \left(\sum_{j=1}^{K}|\hat{g}_{ji,c}|^2 p_{j,c}^{\rm UL} + \sigma^2\right)\left|\delta_{i,c}\right|^2. \end{aligned} \quad (13)$$

By fixing the weights $\omega_{i,c}^{\rm UL}$'s, $\omega_{i,c}^{\rm DL}$'s, the DL transmit beamformer $\mathbf{w}_{i,c}^{\rm DL}$'s, and the UL transmit power $p_{i,c}^{\rm UL}$'s, the receive beamformer for MS-i at the BS on CH-c can be easily obtained as the Wiener filter (MMSE receiver) as

$$\begin{aligned}\mathbf{v}_{i,c}^{\star} &= \arg\min_{\mathbf{v}_{i,c}} \, e_{i,c}^{\mathrm{UL}} \quad (14)\\ &= \left(\sum_{j=1}^{K} p_{j,c}\mathbf{g}_{j,c}\mathbf{g}_{j,c}^{H} + \left[\sigma^2 + \sum_{j=1}^{K}\|\mathbf{w}_{j,c}\|^2\hat{\sigma}^2\right]\mathbf{I}\right)^{-1}\sqrt{p_{i,c}^{\mathrm{UL}}}\mathbf{g}_{i,c},\end{aligned}$$

whereas the receive equalizer at MS-i is the MMSE equalizer as

$$\begin{aligned}\delta_{i,c}^{\star} &= \arg\min_{\delta_{i,c}} \, e_{i,c}^{\mathrm{DL}} \quad (15)\\ &= \frac{\mathbf{w}_{i,c}^{H}\mathbf{h}_{i,c}}{\sum_{j=1}^{K}\left|\mathbf{h}_{i,c}^{H}\mathbf{w}_{j,c}\right|^2 + \sum_{j=1}^{K}|\hat{g}_{ji,c}|^2 p_{j,c}^{\mathrm{UL}} + \sigma^2}.\end{aligned}$$

Then, by fixing $\mathbf{w}_{i,c}^{\mathrm{DL}}$'s, $p_{i,c}^{\mathrm{UL}}$'s, $\mathbf{v}_{i,c}$'s and $\delta_{i,c}$'s, the optimal weight $\omega_{i,c}^{\mathrm{UL}}$ can be obtained by solving the unconstrained optimization

$$\begin{aligned}\omega_{i,c}^{\mathrm{UL}\star} &= \arg\min_{\omega_{i,c}^{\mathrm{UL}}}\left[\omega_{i,c}^{\mathrm{UL}}e_{i,c}^{\mathrm{UL}} - \log\omega_{i,c}^{\mathrm{UL}}\right]\\ &= \left(e_{i,c}^{\mathrm{UL}}\right)^{-1}\\ &= 1\Big/\left(1 - \sqrt{p_{i,c}^{\mathrm{UL}}}\mathbf{v}_{i,c}^{H}\mathbf{g}_{i,c}\right). \quad (16)\end{aligned}$$

Similarly, the optimal weight $\omega_{i,c}^{\mathrm{DL}}$ can be found as

$$\omega_{i,c}^{\mathrm{DL}\star} = \left(e_{i,c}^{\mathrm{DL}}\right)^{-1} = 1\Big/\left(1 - \delta_{i,c}\mathbf{h}_{i,c}^{H}\mathbf{w}_{i,c}\right). \quad (17)$$

Finally, by fixing all other variables, we attempt to optimize the UL transmit powers $p_{i,c}^{\mathrm{UL}}$'s and the DL transmit beamformers $\mathbf{w}_{i,c}$'s. Combining with the expressions in (12) and (13), the objective function in (11) can be reorganized as follows:

$$\begin{aligned}&\sum_{i=1}^{K}\sum_{c=1}^{2}\Big[a_{i,c}\left(\omega_{i,c}^{\mathrm{DL}}e_{i,c}^{\mathrm{DL}} - \log\omega_{i,c}^{\mathrm{DL}}\right) + a_{i,\hat{c}}\left(\omega_{i,c}^{\mathrm{UL}}e_{i,c}^{\mathrm{UL}} - \log\omega_{i,c}^{\mathrm{UL}}\right)\Big]\\ &= \sum_{i=1}^{K}\sum_{c=1}^{2}\Big[p_{i,c}^{\mathrm{UL}}\sum_{j=1}^{K}a_{j,\hat{c}}\omega_{j,c}^{\mathrm{UL}}\left|\mathbf{g}_{i,c}^{H}\mathbf{v}_{j,c}\right|^2\\ &\qquad - \sqrt{p_{i,c}^{\mathrm{UL}}}a_{i,\hat{c}}\omega_{i,c}^{\mathrm{UL}}\left(\mathbf{v}_{i,c}^{H}\mathbf{g}_{i,c} + \mathbf{g}_{i,c}^{H}\mathbf{v}_{i,c}\right)\\ &\qquad + p_{i,c}^{\mathrm{UL}}\sum_{j=1}^{K}a_{j,c}\omega_{j,c}^{\mathrm{DL}}\left|g_{ij,c}\right|^2\left|\delta_{j,c}\right|^2\Big]\\ &+ \sum_{i=1}^{K}\sum_{c=1}^{2}\Big[\sum_{j=1}^{K}a_{j,c}\omega_{j,c}^{\mathrm{DL}}\left|\mathbf{w}_{i,c}^{H}\mathbf{h}_{j,c}\right|^2\left|\delta_{j,c}\right|^2\\ &\qquad - a_{i,c}\omega_{i,c}^{\mathrm{DL}}\left(\delta_{i,c}\mathbf{h}_{i,c}^{H}\mathbf{w}_{i,c} + \delta_{i,c}^{*}\mathbf{w}_{i,c}^{H}\mathbf{h}_{i,c}\right)\\ &\qquad + \|\mathbf{w}_{i,c}\|^2\sum_{j=1}^{K}a_{j,\hat{c}}\omega_{j,c}^{\mathrm{UL}}\|\mathbf{v}_{j,c}\|^2\hat{\sigma}^2\Big]\\ &+ \sum_{i=1}^{K}\sum_{c=1}^{2}\left(a_{i,c}\omega_{i,c}^{\mathrm{DL}}\|\mathbf{v}_{i,c}\|^2 + a_{i,\hat{c}}\omega_{i,c}^{\mathrm{DL}}\left|\delta_{i,c}\right|^2\right)\sigma^2\\ &+ \sum_{i=1}^{K}\sum_{c=1}^{2}\Big[a_{i,c}\left(\omega_{i,c}^{\mathrm{DL}} - \log\omega_{i,c}^{\mathrm{DL}}\right) + a_{i,\hat{c}}\left(\omega_{i,c}^{\mathrm{UL}} - \log\omega_{i,c}^{\mathrm{UL}}\right)\Big]. \quad (18)\end{aligned}$$

By omitting known terms, we then can decompose the above objective function and solve the power allocation problem at each MS and the beamforming design problem at the BS *separately*. More specifically, at MS-i, we attempt to solve the UL power allocation problem

$$\begin{aligned}\underset{p_{i,c}^{\mathrm{UL}}}{\text{minimize}} \;& \sum_{c=1}^{2}\Big[p_{i,c}^{\mathrm{UL}}\sum_{j=1}^{K}a_{j,\hat{c}}\omega_{j,c}^{\mathrm{UL}}\left|\mathbf{g}_{i,c}^{H}\mathbf{v}_{j,c}\right|^2\\ &\qquad - \sqrt{p_{i,c}^{\mathrm{UL}}}a_{i,\hat{c}}\omega_{i,c}^{\mathrm{UL}}\left(\mathbf{v}_{i,c}^{H}\mathbf{g}_{i,c} + \mathbf{g}_{i,c}^{H}\mathbf{v}_{i,c}\right)\\ &\qquad + p_{i,c}^{\mathrm{UL}}\sum_{j=1}^{K}a_{j,c}\omega_{j,c}^{\mathrm{DL}}\left|g_{ij,c}\right|^2\left|\delta_{j,c}\right|^2\Big] \quad (19)\\ \text{subject to } \;& p_{i,1}^{\mathrm{UL}} + p_{i,2}^{\mathrm{UL}} \le p_{i,\max}^{\mathrm{UL}}.\end{aligned}$$

Whereas, at the BS, the optimal DL beamformer can be found by solving

$$\begin{aligned}\underset{\mathbf{w}_{i,c}^{\mathrm{DL}}}{\text{minimize}} \;& \sum_{i=1}^{K}\sum_{c=1}^{2}\Big[\sum_{j=1}^{K}a_{j,c}\omega_{j,c}^{\mathrm{DL}}\left|\mathbf{w}_{i,c}^{H}\mathbf{h}_{j,c}\right|^2\left|\delta_{j,c}\right|^2\\ &\qquad - a_{i,c}\omega_{i,c}^{\mathrm{DL}}\left(\delta_{i,c}\mathbf{h}_{i,c}^{H}\mathbf{w}_{i,c} + \delta_{i,c}^{*}\mathbf{w}_{i,c}^{H}\mathbf{h}_{i,c}\right)\\ &\qquad + \|\mathbf{w}_{i,c}\|^2\sum_{j=1}^{K}a_{j,\hat{c}}\omega_{j,c}^{\mathrm{UL}}\|\mathbf{v}_{j,c}\|^2\hat{\sigma}^2\Big] \quad (20)\\ \text{subject to } \;& \sum_{i=1}^{K}\left[\|\mathbf{w}_{i,1}\|^2 + \|\mathbf{w}_{i,2}\|^2\right] \le p_{0,\max}^{\mathrm{DL}}.\end{aligned}$$

We note that problem (19) is a convex quadratic program in $\sqrt{p_{i,c}^{\mathrm{UL}}}$'s. Its optimal solution can be easily obtained in closed-form at the top of the next page in Equation (21), where $\lambda_i^{\mathrm{UL}} \ge 0$ is the Lagrangian multiplier associated with the power constraint at MS-i. The optimal solution of λ_i^{UL} can be easily found by the bisection method. Similarly, problem (20) is convex in $\mathbf{w}_{i,c}$'s. Its optimal solution can also be obtained in closed-form as presented in Equation (22), where $\lambda^{\mathrm{DL}} \ge 0$ is the Lagrangian multiplier to enforce the power constraint at the BS.

By iteratively updating $\mathbf{v}_{i,c}$'s, $\delta_{i,c}$'s, $\omega_{i,c}^{\mathrm{UL}}$, $\omega_{i,c}^{\mathrm{DL}}$, $p_{i,c}^{\mathrm{UL}}$, and $\mathbf{w}_{i,c}$'s, we obtain the weighted MMSE algorithm to jointly maximize DL and UL sum-rate, as summarized in Algorithm 1. The convergence of the algorithm is guaranteed due to the monotonic decrease of the objective function in problem (11) after each iteration.

IV. Sum-rate Maximization with Dynamic Channel Assignment

In this section, we investigate the next step in dynamically allocate the UL and DL channel assignments for each MS. With the obtained data rate for both DL and UL transmissions given in Section III, we consider the following optimization to find the channel assignment for each MS:

$$\begin{aligned}\underset{a_{i,c}}{\text{maximize}} \;& \sum_{i=1}^{K}\sum_{c=1}^{2}a_{i,c}\left[\log\left(1+\gamma_{i,c}^{\mathrm{DL}}\right) + \log\left(1+\gamma_{i,\hat{c}}^{\mathrm{UL}}\right)\right] \quad (23)\\ \text{subject to } \;& a_{i,1} + a_{i,2} = 1, \forall i,\\ & a_{i,1}, a_{i,2} \in [0,1], \forall i.\end{aligned}$$

While problem (23) is a nonconvex integer program, its globally optimization can be found. By relaxing the last constraint into $0 \le a_{i,c} \le 1$, one has a linear program (LP)

$$\sqrt{p_{i,c}^{\mathrm{UL}\star}} = \frac{a_{i,\hat{c}}\omega_{i,c}^{\mathrm{UL}}\mathbf{g}_{i,c}^{H}\mathbf{v}_{i,c}}{\sum_{j=1}^{K}\left[a_{j,\hat{c}}\omega_{j,c}^{\mathrm{UL}}\left|\mathbf{g}_{i,c}^{H}\mathbf{v}_{j,c}\right|^{2}+a_{j,c}\omega_{j,c}^{\mathrm{DL}}\left|g_{ij,c}\right|^{2}\left|\delta_{j,c}\right|^{2}\right]+\lambda_{i}^{\mathrm{UL}}}. \tag{21}$$

$$\mathbf{w}_{i,c}^{\star} = \left(\sum_{j=1}^{K}\left[\omega_{j,c}^{\mathrm{DL}}|\delta_{j,c}|^{2}\mathbf{h}_{j,c}\mathbf{h}_{j,c}^{H} + a_{j,\hat{c}}\omega_{j,c}^{\mathrm{UL}}\|\mathbf{v}_{j,c}\|^{2}\hat{\sigma}^{2}\mathbf{I}\right] + \lambda^{\mathrm{DL}}\mathbf{I}\right)^{-1}\mathbf{h}_{i,c}\delta_{i,c}^{*}\omega_{i,c}^{\mathrm{DL}}a_{i,c}. \tag{22}$$

Algorithm 1: Weighted MMSE for DL and UL Sum-rate Maximization

Input: Starting points for $\mathbf{w}_{i,c}$'s and $p_{i,c}^{\mathrm{UL}}$'s;
If no starting points are given, initialize by randomizing $\mathbf{w}_{i,c}$'s such that $\sum_{i=1}^{K}\sum_{c=1}^{2}\|\mathbf{w}_{i,c}\|^2 = p_{0,\max}^{\mathrm{DL}}$ and $p_{i,c}^{\mathrm{UL}}$'s such that $\sum_{c=1}^{2} p_{i,c}^{\mathrm{UL}} = p_{i,\max}^{\mathrm{UL}}$;
repeat
    Set $\bar{\mathbf{w}}_{i,c} \leftarrow \mathbf{w}_{i,c}$;
    Set $\bar{p}_{i,c}^{\mathrm{UL}} \leftarrow p_{i,c}^{\mathrm{UL}}$;
    for $i = 1, \ldots, K$ **do**
        Update the UL receiver $\mathbf{v}_{i,c}$ as in (14);
        Update the DL equalizer $\delta_{i,c}$ as in (15);
        Update the UL weight $\omega_{i,c}^{\mathrm{UL}}$ as in (16);
        Update the DL weight $\omega_{i,c}^{\mathrm{DL}}$ as in (17);
    end
    for $i = 1, \ldots, K$ **do**
        Update the UL transmit power $p_{i,c}^{\mathrm{UL}}$ as in (21);
    end
    Update the DL beamformer $\mathbf{w}_{i,c}$ as in (22);
until *convergence*;

in $a_{i,c}$'s, whose optimal solution can be found efficiently by standard convex optimization techniques [14]. Interesting, the obtained optimal solution of the LP lies at a vertex where $a_{i,c}$ will be either 0 and 1. Thus, given $\gamma_{i,c}^{\mathrm{DL}}$'s and $\gamma_{i,c}^{\mathrm{UL}}$, solving the LP will result in the optimal channel assignment for each user. However, once $a_{i,c}$'s are set to 0 or 1, the channel assignment for each user will be fixed and cannot be optimized anymore. To circumvent this issue, we adopt the gradient method to "slowly" update the channel assignment variable $a_{i,c}$'s [11]. More specifically, given that $r_{i,c} \triangleq \log\left(1+\gamma_{i,c}^{\mathrm{DL}}\right) + \log\left(1+\gamma_{i,\hat{c}}^{\mathrm{DL}}\right)$ is the gradient of $a_{i,c}$, one can apply the following gradient projection method

$$\mathbf{a}_i = \mathcal{P}_{\mathcal{S}_i}\left\{\mathbf{a}_i + \alpha_n \mathbf{r}_i\right\}, \tag{24}$$

where $\mathbf{a}_i = [a_{i,1}, a_{i,2}]^T$, $\alpha_n \overset{n\to\infty}{\longrightarrow} 0$ is a small diminishing step-size, $\mathbf{r}_i = [r_{i,1}, r_{i,2}]^T$, and $\mathcal{S}_1 \triangleq \{\mathbf{a}_i \in \mathbb{R}^2 : \mathbf{a}_i \geq \mathbf{0}, a_{i,1} + a_{i,2} \leq 1\}$. We summarize the combination of update $a_{i,c}$'s and Algorithm 1 in Algorithm 2.

Proposition 2. *The iterative procedure presented in Algorithm 2 converges to a locally optimal solution of problem* (9).

Proof: Given $a_{i,c}$'s and the starting points for $\mathbf{w}_{i,c}$'s and $p_{i,c}^{\mathrm{UL}}$, Algorithm 1 will converge monotonically to a locally optimal solution of the MSE minimization problem (11). Per Proposition 1, this solution is also locally optimal to the sum-rate maximization problem (10). Thus, the sum UL and DL data rate will be improved by Algorithm 1. The gradient update of $a_{i,c}$'s will further improve this sum-rate. Thus, Algorithm 2 will monotonically increase the the objective function of problem (9), which leads to a locally optimal solution. ■

Algorithm 2: DL and UL Sum-rate Maximization with Dynamic Channel Selection

Initialize $a_{i,1} = a_{i,2} = 1/2$, $\forall i$;
repeat
    Call Algorithm 1;
    Utilize the updated $\mathbf{w}_{i,c}$'s and $p_{i,c}^{\mathrm{UL}}$'s for the next iteration;
    Update $\mathbf{a}_i$'s as in (24);
until *convergence*;

V. Simulation Results

TABLE I
SIMULATION PARAMETERS AND SETTINGS

Parameter	Value
Carrier frequency f_c	2 GHz
AWGN power	−90 dBm
BS transmit power	40 dBm
MS transmit power	33 dBm
MS-BS distance	250 m
BS height	30 m
MS height	1.5 m

In this section, we present the simulation results to compare the sum-rate of a MU-MISO system operating in the FD and HD modes. When the system is in the FD mode, Algorithm 2 is applied to dynamically select the channel for each MS and optimize the DL/UL transmission rates. When the system is in the HD mode, the better rate obtained with $\mathcal{S}_1 = \mathcal{K}$ or $\mathcal{S}_2 = \mathcal{K}$ is then chosen. We assume that the BS is equipped with $M = 4$ transmit/receive antennas and located at the center of the network. The MSs are uniformly located in a circle, centered at the BS with a radius of 250 m. Based on the distance between each pair of transmitter and receiver, the large-scale fading is generated using the COST231 model [15]. The simulation parameters, which mimic the LTE-Advanced physical layer, are summarized in Table I.

Fig. 2 displays the achievable system sum-rate versus the RSI level at the BS. Herein, the RSI power in dBm is defined as $\hat{\sigma}^2 p_{0,\max}^{\mathrm{UL}}$, i.e., when the BS transmits at its maximum power. Note that the HD mode is not affected by the RSI. Thus, the system sum-rate in the HD mode remains unchanged, at above 20 b/s/HZ, regardless of the RSI level. As observed

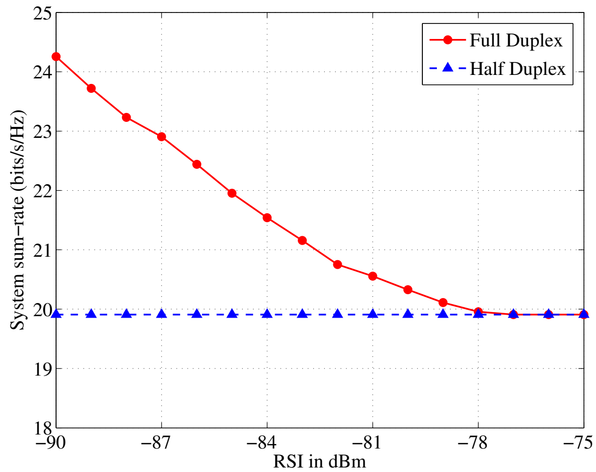

Fig. 2. System sum-rate in FD and HD modes *versus* RSI at the BS with $K = 4$ MSs.

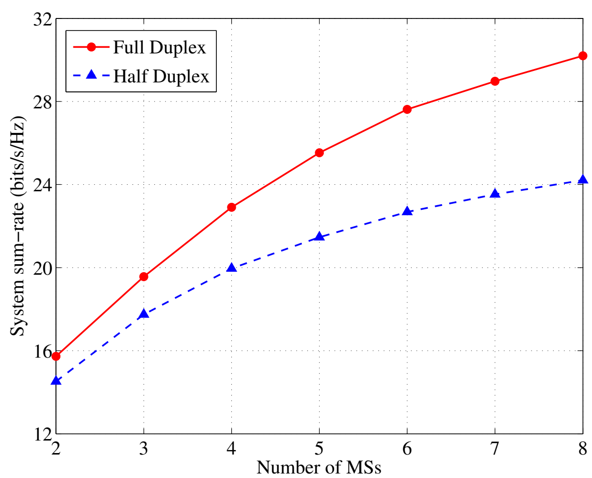

Fig. 3. System sum-rate in FD and HD modes *versus* number of MSs K with the RSI at the BS at -87 dBm.

from the figure, when the RSI is low (at around AWGN floor), operating in the FD mode can improve the data rate by about 20%. This performance advantage reduces and diminishes when the RSI reaches to -78 dBm. Note that the proposed Algorithm 2, which optimizes the channel assignment, will switch to the half-duplex mode at high RSI. This explains the overlapping part of the curves in Fig. 2.

Fig. 3 displays the achievable system sum-rate versus the number of MSs K in the system with the RSI at the BS being set at -87 dBm. Note that the BS can support at most 4 UL degrees of freedom (DoF) and 4 DL DoF in the HD mode with 4 transmit and 4 receive antennas. When the number of MSs K exceeds 4, the system data rate in the HD keeps increasing due to the multiuser diversity. On the contrary, the BS can potentially support 8 UL DoF and 8 DL DoF in the FD mode (4 DoF of each channel). Thus, the system sum-rate increases at a faster rate with K, when being in the FD mode than being in the HD mode. As observed in the figure, the FD mode provides about 35% data rate improvement at $K = 8$. However, in both simulations presented in Figs. 2 and 3, the FD mode does not double the data-rate relatively the HD mode, which is achievable in one-to-one communication systems [2]. This result can be explained by the limiting factors in a FD multiuser system: the RSI at the BS and the additional co-channel interference at the MSs caused by one group's UL transmission to the other group's DL reception.

VI. Conclusion

In this work, we have proposed a framework to optimize the channel assignment and maximize the UL and DL sum data-rate in a FD multiuser system. Since the joint optimization problem is a difficult nonconvex problem, we have proposed an iterative algorithm to obtain at least a locally optimal solution. Specifically, we have considered a gradient method to update the channel assignment and the weighted MMSE algorithm to maximize the system sum-rate. Simulation results show that the FD mode has to the potential to enhance a multiuser system's data-rate as much as 35%, compared to the HD mode.

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