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Letters

Closed-Form Multipole Debye Model for Time-Domain Modeling of Lossy Dielectrics

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***Abstract*—Lossy dielectrics in printed circuit boards and integrated circuit packages can be represented by using a Debye model. This allows accurate signal and power integrity analysis, which depends on the accuracy of material properties of the board or package. Such a Debye model needs multiple poles for accurate representation of the loss tangent over a broad frequency range. Electromagnetic and circuit simulations can then include the impact of frequency-dependent dielectric constant and loss. In this letter, we present an efficient and closed-form multipole Debye model, automating the modeling of lossy dielectrics for inclusion in time-domain electromagnetic or circuit simulators.**

***Index Terms*—Causality, complex permittivity, Debye model, dielectric constant, loss tangent.**

I. Introduction

LOSSY dielectrics in printed circuit boards and integrated circuit packages result in a complex permittivity given by

$$\varepsilon = \varepsilon_r(1 - j\tan\delta)\varepsilon_0 \tag{1}$$

where ε_r is the dielectric constant, $\tan\delta$ is the loss tangent, and ε_0 is the permittivity of free space. Kramers–Kronig relations for dielectrics dictate that for lossy substrates (i.e., when $\tan\delta > 0$), ε_r will be frequency dependent. Therefore, for broadband signal and power integrity simulations, modeling a lossy substrate with a constant dielectric constant can result in errors. For time-domain simulations, such a modeling error can even create an unstable model response.

A physical model to represent the frequency-dependent variation of the dielectric constant and loss tangent can be obtained by using a Debye model as

$$\varepsilon = \varepsilon_\infty + \sum_{n=0}^{N-1} \frac{a_n}{s + b_n} \tag{2}$$

where a_n and b_n are physically related to the strength and time constants of various relaxation processes, ε_∞ is the high-frequency asymptotic value of the permittivity, and s is the Laplace variable. The order of the approximation N can be chosen as high as possible as long as the extracted a_i and b_i are all positive coefficients. This ensures that the obtained model represents a passive network over all frequencies and, therefore, satisfies Kramers–Kronig relations for dielectrics.

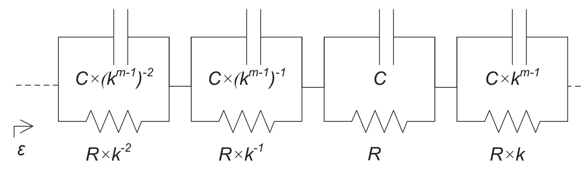

Fig. 1. Infinite Debye model representing a lossy dielectric.

A commonly used Debye model is obtained by using infinite number of poles (i.e., as N approaches infinity) [1]. However, a finite-pole Debye model as presented in (2) can be easily integrated in time-domain electromagnetic simulators (e.g., based on the finite-difference time-domain method), or in circuit simulators for transient analysis [2].

For substrates with a constant loss tangent, the complex permittivity in (1) is a network function with a constant argument [3], [4] given by

$$\varepsilon = a s^{-2\delta/\pi}\varepsilon_0 \tag{3}$$

where a is an arbitrary positive constant. Substituting $s = j\omega$ indeed yields a complex permittivity in a form similar to (1)

$$\varepsilon = a(j\omega)^{-2\delta/\pi}\varepsilon_0 = a\omega^{-2\delta/\pi}\cos\delta(1 - j\tan\delta)\varepsilon_0. \tag{4}$$

The complex permittivity in (3) is a nonrational function; hence, a multipole Debye model in (2) is an approximate *RC*-network realization of this complex permittivity. The Debye model cannot be obtained by a straightforward application of the vector-fitting algorithm [5], since it does not guarantee positive coefficients in (2). An efficient *RC* model for interconnect capacitance has been obtained in [6], [7] and [8] based on the approximate model for an *RC* constant-argument driving-point admittances in [9]. An alternative infinite *RC* network representation for constant-argument driving-point impedances is given in [10], which can be converted exactly to the form presented in [9] after a change of variables.

In this letter, we introduce a closed-form multipole Debye model for the first time for substrates with a constant loss tangent.

II. Multipole Debye Model

The multipole Debye model in (2) can be interpreted as the impedance of an *RC* network. For constant-argument driving-point impedances as in (3), an infinite *RC* network representation can be found following the approach presented in [9]. The resulting network is shown in Fig. 1, where $m = \pi/2\delta$ and k is a spacing factor. Truncating this infinite network results in the Debye model in (2).

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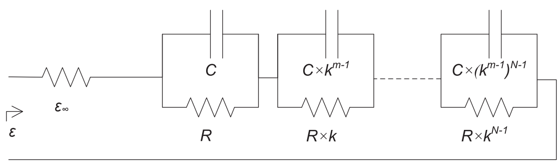


Fig. 2. Debye model in (2) of order N representing a lossy dielectric.

The first truncation can be made for all *RC* branches to the left of the branch with the resistance R. At lower frequencies, these branches start behaving mostly resistive because of the higher impedances associated with the capacitors. We can then approximate the contribution of these branches by adding all the resistances in series to obtain the total resistance of this truncated part as

$$\varepsilon_\infty = Rk^{-1}(1 + k^{-1} + k^{-2} + \cdots) = \frac{R}{k-1} \tag{5}$$

which provides us the truncated Debye model in Fig. 2.

In Fig. 2, the resistances are calculated from $R = \varepsilon_\infty(k-1)$. The capacitances are obtained from $C = 1/(R\omega_0)$ [9], where ω_0 corresponds to the upper frequency bound for the validity of the model. The number of *RC* branches included in the multipole Debye model determine the bandwidth of the model. From the equivalent circuit model in Fig. 2, we can finally obtain the coefficients of the Debye model in a closed form as

$$\varepsilon = \varepsilon_\infty + \sum_{n=0}^{N-1} \frac{\varepsilon_\infty(k-1)k^n}{1 + sk^{nm}/\omega_0}. \tag{6}$$

As an example, Fig. 3 shows the variation of dielectric constant and loss tangent for $\varepsilon_\infty = 4\varepsilon_0$, $\tan\delta = 0.02$, $N = 10$, and $\omega_0 = 10^{10}$. The spacing parameter k determines the accuracy of the model. Using a larger k improves the bandwidth of the model at the expense of larger oscillations. Fig. 3 shows how the bandwidth of the model is increased by changing k from 1.02 to 1.03. A critical observation in this figure is the rounding-off of the edges of the loss tangent. For example, at around 10 GHz, the $k = 1.02$ case starts approaching zero, hence loses its accuracy, before the $k = 1.03$ case. This implies that the accuracy can be increased by using a smaller spacing parameter k; however, the order of the model N may need to be increased to compensate for the rounding-off of the edges of the loss tangent.

Within the frequency range where the model is accurate, oscillations around the correct value can be observed, especially for the $k = 1.03$ case in Fig. 3(b). Each cycle in these oscillations corresponds to an *RC* term in the summation of (6). These cycles are completed at logarithmically evenly spaced frequencies, with a ratio of k^m between the frequencies of two successive cycles [9]. If $k^m = 10^{1/d}$, for example, adding d elements in the Debye model would increase the bandwidth of the model by a decade. Since d provides a more intuitive description of the model bandwidth, in the following, we assume d will be the actual input by the user instead of k.

In a typical practical case, the Debye model will be based on the loss tangent $\tan\delta$ and dielectric constant ε_r provided at the center frequency ω_c. We assume that the order N and the elements required per decade d are also provided. The center frequency will be associated with the middle term in the summation of (6), and it is desirable to have an accurate Debye model at frequencies extending a few decades lower and higher than the center frequency.

Based on the input parameters of $\varepsilon_r, \tan\delta, \omega_c, N$, and d, the coefficients in the Debye model (6) will be obtained as follows. The spacing parameter can be obtained from $m = \pi/2\delta$ as

$$k = 10^{1/md}. \tag{7}$$

The upper frequency bound ω_0 can be related to the center frequency of the bandwidth ω_c as

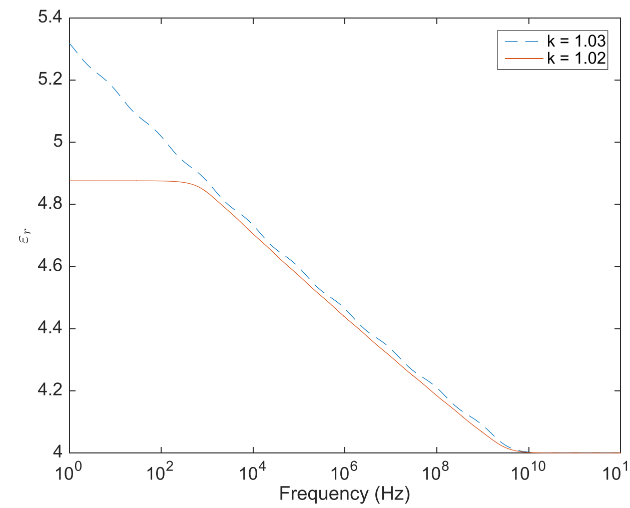
$$\omega_0 = \omega_c 10^{\frac{N-1}{2d}} \tag{8}$$

since there are $(N-1)/2$ terms increasing the bandwidth towards higher frequencies from the center frequency at ω_c.

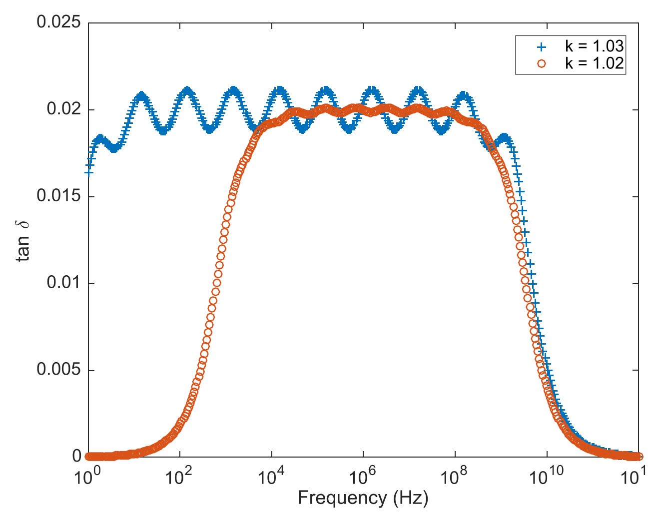
Finally, the asymptotic value of the permittivity at very high frequencies can be calculated as

$$\varepsilon_\infty = \frac{2\varepsilon_r\varepsilon_0}{k^{\frac{N-1}{2}}(k+1)} \tag{9}$$

by expanding the infinite series calculation in (5) to the center branch and considering that the center branch is transitioning at ω_c [9] and contributes only half of its resistance to the overall resistance.

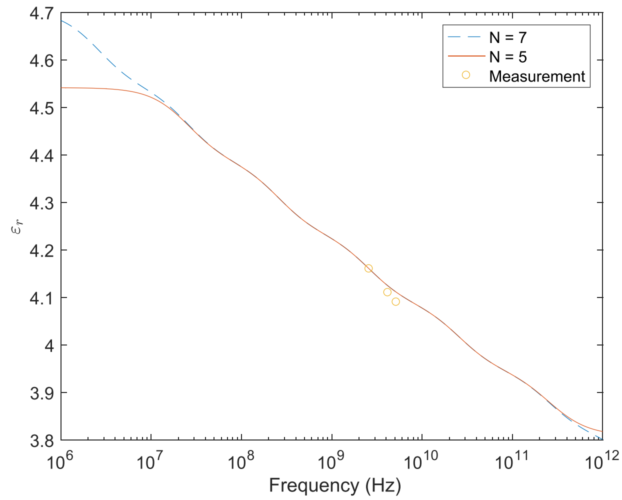


(a)

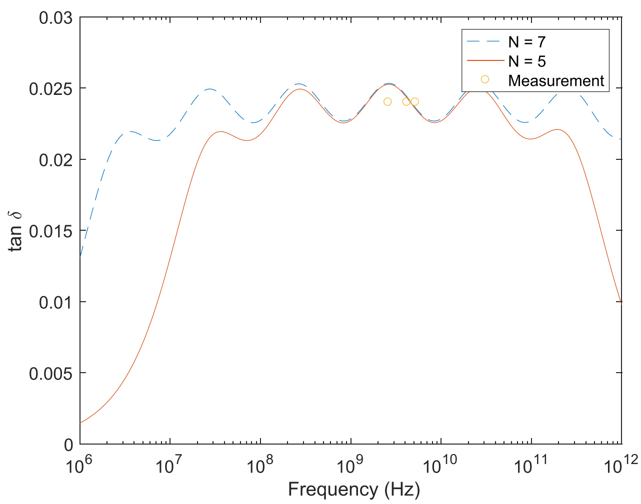


(b)

Fig. 3. Spacing parameter k determines the accuracy of the model. (a) Dielectric constant. (b) Loss tangent for $\varepsilon_\infty = 4\varepsilon_0$, $\tan\delta = 0.02$, $N = 10$, and $\omega_0 = 10^{10}$.



(a)



(b)

Fig. 4. (a) Dielectric constant. (b) Loss tangent for $\varepsilon_r = 4.16$ at 2.6 GHz, $\tan\delta = 0.024$, and $d = 1$ (one element per decade). Incrementing the order of the model by one increases the bandwidth of the model by a decade.

III. Characterization and Modeling of FR-4

An FR-4 board is characterized in [11] with a dielectric constant of 4.16 and $\tan\delta = 0.024$ at 2.6 GHz, which will be used as the input parameters in the Debye model. Following the methodology in [6] and [12], we also extracted the dielectric constant at the second and third resonant frequency of the measured cavity resonator as $\varepsilon_r = 4.11$ at 4.2 GHz and $\varepsilon_r = 4.09$ at 5.2 GHz. The loss tangent was extracted as $\tan\delta = 0.024$ at all three frequencies. Fig. 4 is based on these parameters and using $d = 1$ (one element per decade). The loss tangent is within $\pm 7\%$ of 0.024 for a bandwidth of two decades using $N = 5$. Each additional element increases the bandwidth of the model by a decade. The model can be made arbitrarily more accurate by using a larger d; however, this would require a larger number of elements for achieving the same bandwidth. Additional simulations have also shown similar accuracy and bandwidth for various dielectric constants or loss tangents.

IV. Conclusion

In this letter, we presented a closed-form multipole Debye model. The model can be made arbitrarily more accurate and broadband by increasing its order. The efficiency of the model can also be increased with a tradeoff of introducing ripples in the dielectric constant and loss tangent. For a typical example, the loss tangent was maintained with an accuracy of $\pm 7\%$ where each additional element in the Debye model increases the bandwidth by a decade.

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